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By *J. ROBERTSON*,

Teacher of the MATHEMATICS.

LONDON:

Printed for J. WILCOX, at *Virgil's Head*, over-against the *New Church* in the *Strand*; and J. HODGES, at the *Looking-glass* on *London Bridge*.

MDCCXXXIX.

2

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THE
PREFACE.

AMONG *the ſeveral Branches of the Mathematics taught by me, that of* Menſuration *has very often occurred; when finding none of the Books on that Subject but what were defective, as alſo not ſo methodical as I could have wiſh'd, I was thereby induced to compoſe this new Syſtem for the Uſe of my Pupils, which I apprehend will be found not only fully to anſwer the Title, but likewiſe an Introduction to the higher Branches of the Mathematics.*

I preſume the Reader to be previouſly acquainted with Addition, Subtraction, Multiplication, Diviſion, *and* Reduction in whole Numbers, [*or* Integers] *by which he will be able to comprehend with Eaſe, all that this Book contains.*

As an Introduction I have given the Doctrine of Decimal Arithmetic, *wherein I have not only handled the Rules commonly given in the Books of Menſuration; but have alſo fully treated of the Doctrine of* Single Repetends *(or* Circulating Numbers) *a Thing ſo uſeful in Meaſuring, that I wonder ſo many Books on that Subject have been wrote without conſidering them: For it very often happens, that the Dimenſions of Superficies and Solids will not be either* 3, 6, *or* 9 *Inches (whoſe Decimals are Terminate) which are generally the Inches annexed to Feet in the Examples given in the Books of Meaſuring, when the ſame is to be wrought by Decimals.*

I have purpoſely omitted the Doctrine of Compound Repetends, *they ſeldom occurring in common Menſuration; but thoſe who are willing to known more of that Subject, I refer them to the late ingenious Mr* CUN's Treatiſe of Fractions, *or to a Treatiſe now in the Preſs, intitled* The Compendious Aſtronomer, *written by my Friend and Acquaintance Mr* C. Brent, *where the whole Theory of Decimals is amply explained and Demonſtrated.*

To this I have added the Extraction of the Square *and* Cube Root, *with the Explanation of the common Chracters (or Signs) uſed in Mathematical Books, to ſhorten the Work.*

In the Firſt Part of the Meaſuring *I have fully treated of the* Menſuration of all Superficial Figures, *as far as a* Right Line *and a* Circle *are concerned;*

concerned; and herein I have observed the following Order:

Firſt, *To give ſuch Definitions as are neceſſary to conceive the Form of plain Figures.*

Secondly, *Three Geometrical Problems.*

Thirdly, *From two Propoſitions to deduce the Meaſurement of ſuch Sort of Figures, as moſt commonly occur to the ſeveral Sorts of Artificers concerned in Building; and herein I have ſhewn how to operate by Duodecimals (commonly called* Croſs Multiplication) *in a more eaſy Manner than any yet publiſhed. And have wrought moſt of the Examples, both by* Duodecimals *and* Decimals, *whereby the Methods may be compared.*

And Fourthly, *I have given Rules and proper Examples whereby any other Right-lined Figure may be meaſured; and the* Circle *I have treated of in a different (though I believe more eaſy) Manner than is to be met with in any other practical Author; and the ſeveral Rules for the different Segments of a Circle and other Things pertaining to a Circle, I have illuſtrated by a ſufficient Number of eaſy, uſeful, and entertaining Examples.*

In the Second Part of the Meaſuring *I have fully treated of the Meaſurement of ſuch Solid Figures, as are bounded by Right-lined and Circular Superficies; and herein I have alſo obſerved the following Order:*

Firſt, *To give ſuch Definitions as are neceſſary to conceive the Form of Solid Figures.*

Secondly, *To shew the common Methods used in the Measurement of the several Sorts of Timber, with their Errours, and also how Timber may be expeditiously measured without any sensible Loss.*

And Thirdly, *How to find the Solidities and Superficies of various Solids, such as often occur to Workmen in the ordinary Course of their Business, which I have exemplified by various Examples suited to the several Rules and Cases.*

To which I have added an Appendix, *wherein is shewn the Measurement of the* Conic Sections, *and the Solids generated by their Rotation on their Axes; all which is sufficiently explained by Definitions, Rules, and Examples. And here the Business of measuring the Superficial and Solid Contents of* Groin Arches *is performed, which is made clear and easy by proper Rules and Familiar Examples; together with the Mensuration of the Superficial and Solid Contents of the* Regular Solids, *with proper Tables to facilitate all their Operations: And a Table of the* Specific Gravity of Metals *and* Other Bodies *with it's Uses. Also* Decimal Tables *of* Coin, Weights, Measures, *and* Time, *with their Construction and Uses.*

The whole is illustrated with Variety of useful and pleasant Examples, so calculated and contrived, as to exercise the foregoing Propositions, and also adapted to such Uses as often occur in Life. Together with three Copper Plates, wherein are neatly engraven the various Kinds of Figures referred

ferred to in the Definitions and Examples. I have all the Way through the several Parts of the Practical Menſuration, *ſought the Values of the Contents of the ſeveral Sorts of Work, and that by the moſt ready Methods of* Aliquot Parts *and* Decimals; *which I conceive will be of no ſmall Uſe to the Workman. And I have alſo in their proper Places given the Allowances and Cuſtoms uſed amongſt the* Builders *and* Meaſurers.

I have omitted the Demonſtrations of moſt of the Rules, eſpecially thoſe that are immediately demonſtrated from Geometry *or* Algebra; *as knowing that thoſe Sort of Readers, for whom this Book is chiefly calculated, would not with Eaſe underſtand them, and therefore would rather ſerve to puzzle and amuſe, than inſtruct them, (beſides the ſwelling of the Book beyond it's intended Limits). And I conceive that they who would underſtand them, have no Occaſion to look into a Book of pure Menſuration, for the learning that Science, which they cannot help being acquainted with in the ſtudying thoſe Parts of the Mathematics, that are requiſite for the Demonſtration of the Rules of Menſuration. And thoſe who would be ſatisfied of the Truth of the Rules in the enſuing Treatiſe, may find them abſtractedly demonſtrated in many Mathematical Authors.*

I have alſo omitted ſhewing the Uſe of the Sliding Rule, *becauſe I think at beſt it is but gueſſing at the Anſwers required; but thoſe who would be acquainted therewith, may find it in a*

Multitude

Multitude of Mathematical Books, many whereof are written for that Purpose only.

I shall now conclude, with wishing the Reader may find as much Pleasure and Satisfaction in perusing this Treatise, as I did in compiling it; not doubting thereof, gives an additional Pleasure to his Friend and

September
27, 1738.

humble Servant,

John Robertson.

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ADVERTISEMENT.

ARITHMETIC, MENSURATION, GEOMETRY, ALGEBRA, TRIGONOMETRY, NAVIGATION, SURVEYING, FORTIFICATION, GUNNERY, and other useful Branches of the *Mathematics*, are taught after an easy and expeditious Manner by the *AUTHOR*; who may be spoken with, or directed to, at Mr N. ADAMS's, *Optition* to their *Royal Highnesses* the *Prince and Princes of Wales*, near *Northumberland-House* at *Chairing-Cross*.

N. B. The *Author* also surveys Land, and delineateth Maps thereof in the most accurate Manner.

A COMPLEAT
TREATISE
OF
MENSURATION.

SECTION I.

DECIMAL FRACTIONS.

DEFINITIONS.

I. A FRACTION is ſome Part or Parts of an Unit; that is, Unity or 1 is ſuppoſed to be divided into ſome Number of Parts, and a Fraction is one or more of thoſe Parts.

ALL Fractions are generated by Diviſion, thus: When one Number is to be divided by another, and there happens a Remainder, that Remainder ſet over the Diviſor, with a Line drawn between them (in the ſame manner as you would expreſs 1, 2, or

3 Farthings, *viz.* $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, which are the Fractions or Parts of a Penny) constitutes a Fraction, the Value or Worth of which will always be less than an Unit: Thus, if 38 was to be divided by 6, the Quotient would be 6, and something more, *viz.* $6\frac{2}{6}$; for after Division is ended, there remains 2; now if this 2 be set over the Divisor 6, as above directed, it will stand thus $\frac{2}{6}$, which annexed to the Quotient 6, is $6\frac{2}{6}$, and is thus read, six Units and two parts in six of an Unit or 1. Now it is plain, that $\frac{2}{6}$ is less than an Unit, for if it were an Unit, this Unit instead of the Fraction being annexed to the 6 would make 7, which would be a wrong Quotient; for 38 divided by 6 cannot produce 7; therefore the Fraction $\frac{2}{6}$ is less than an Unit.

II. In a Fraction thus constituted, that number above the Line is called the Numerator, and that below the Line is called the Denominator.

III. A Fraction is called Vulgar, when the several Figures in the Denominator added together make a Sum greater than an Unit. Thus, in the Fraction $\frac{1}{2}$ the Denominator 2 is greates than an Unit, and is therefore a Vulgar one. And in the Fraction $\frac{7}{12}$, the two Figures in the Denominator, *viz.* 1 and 2 added together make 3, which is greater than Unity, and the Fraction therefore is a Vulgar one: And in the Fraction $\frac{812}{2689}$, the four Figures in the Denominator, *viz.* 2 and 6 and 8 and 9 added together makes 25, which is a Sum greater than an Unit or 1; and the Fraction is therefore a Vulgar one, and the same in any other.

Hence it will be easy to conceive, that among Vulgar Fractions there may be Denominators consisting of 1, 2, 3, 4; or 10, 16, 100, &c. Places, and may all be significant Digits; which makes the Business of Vulgar Fractions very troublesome.

VI. A

IV. A Fraction is called a Decimal, when the Sum of the ſeveral Characters in the Denominator is equal to an Unit; or thus,

V. A Decimal Fraction, is when the Unit is ſuppoſed divided into ſuch a number of Parts as may be expreſſed by an Unit, with a certain number of Cyphers annexed to it. Hence the fourth Definition is plain.

VI. A Decimal Fraction is expreſſed (or is always wrote) without it's Denominator, and is diſtinguiſhed by prefixing a Point or Comma to the left Hand of the Figures ſaid to denote the Fraction, the Denominator being underſtood always to poſſeſs a Number compoſed of an Unit, with as many Cyphers annexed to the right Hand as the Decimal (that is, the Numerator) hath Figures—Thus ,6 is underſtood to be $\frac{6}{10}$; and ,47 to be $\frac{47}{100}$; and ,763218, to be $\frac{763218}{1000000}$, &c.

Hence ariſeth the ſuperiour Excellency of Decimal Fractions to that of Vulgar ones: For Decimals are always managed without any regard to their Denominators; whereas in Vulgar Fractions there generally ariſeth more Trouble on Account of the Diverſity of the Denominators, than from the Numerators themſelves.

VII. A terminate Decimal, is that which ends at a certain number of Places; but an interminate no where ends.

VIII. A Repetend is a Decimal where the Figures circulate; that is, when the ſame Figure or Figures continually repeat, or run on the ſame.

If one Figure continually circulate, it is called a *ſingle Repetend*: But if more than one Figure circulate, it is called a *compound Repetend*.

IX. The firſt Place next the mark of Diſtinction in any Decimal Expreſſion is called the Place

of Primes; and the following Places are called Seconds, Thirds, Fourths, &c.

THEOREMS.

I. CYPHERS to the right Hand of Decimals neither increase nor decrease their Value; but Cyphers between the separating Point and the Figures of the Decimal diminish the Value; thus ,5 is $\frac{5}{10}$, but ,05 is $\frac{5}{100}$, and ,005 in $\frac{5}{1000}$, &c. and for any other Decimal the same.

II. If the mark of Distinction in any mixed or fractional Expression be moved but one place towards the left Hand, then every Figure, and consequently the whole Expression, is but a tenth part of what it was before: But if it had been removed towards the right Hand, then the whole Expression would be ten times as much as it was before it was removed.

Note, A whole Number with a Fraction annexed thereto, is called a mixed Expression, and a Fraction without a whole Number, is called a fractional Expression.

III. Every terminate Decimal may be considered as interminate, by making Cyphers the Repetend. For (*per Theo.* 1.) they do not alter the Value of the Decimal.

IV. Any decimal Expression may be continued as far as you please, by repeating the circulating Figure or Figures.

Note, Any Number of Decimals thus filled up, so as to have a number of Places equal to any other Decimal, are said to be made conterminous.

V. In all Results, if the Repetend consist of all nines, reject them, and make the next superiour Place an Unit more; thus, 7,23999, &c. is made 7,24, &c.

VI. In

VI. In all circulating Expressions, dash the first and last of the Repetend, omitting the intermediate Places; thus, 4,28333, or 7,843843843 &c.

SECTION II.

REDUCTION,

OR the Methods used to bring any Vulgar Fraction, or an Expression of different Denominations to it's equivalent Decimal Expressions: Or any Decimal Expression to it's Value in different Denominations.

First, To reduce a Vulgar Fraction, to it's equivalent Decimal one.

RULE.

Make the Numerator a Repetend by adding thereto Cyphers, and this divide by the Denominator, and the Quotient is the Decimal sought after; which must always have as many Places as there were repeating Cyphers used—If when Division is ended there happen not so many Figures in the Quotient as there were Cyphers used to gain this Quotient, annex to the left Hand as many Cyphers as there are Places wanting, and this will be the Decimal Expression sought; minding to set before it the Comma, or mark of Distinction.

Reduce $\frac{1}{4}$, and $\frac{1}{2}$, and $\frac{3}{4}$, and $\frac{5}{8}$, and $\frac{7}{12}$, each to their equivalent Decimals.

4)1,00(,25 —— 2)1,0(,5 —— 4)3,00(,75 —
8)5,000(,625 — 12)7,0000(,58333, &c.

ANSWERS.

$\frac{1}{4}$ is ,25. $\frac{1}{2}$ is ,5. $\frac{3}{4}$ is ,75. $\frac{5}{8}$ is ,625. and $\frac{7}{12}$ is ,58333, &c.

By considering the last Example, it will be easy to conceive how Repetends are generated; and also, to see that 3 would continually repeat.

Reduce $\frac{9}{56}$, and $\frac{17}{286}$, and $\frac{217}{2495}$, and $\frac{83}{9761}$, each to their equivalent Decimal.

```
1st. 56) 9,00000000 (,16071428, &c.
          340
           400
            80
            240
             160
              480
               32

2d. 286) 17,0000000 (,0594405
           2700
            1260
             1160
              1600
               170

3d. 2495) 217,00 (,08697394
           17400
            24300
             18450
              9850
              23650
               11950
                1970, &c.
```

4th.

4th. 9768) 83,000 (,0084961097
48560
93880
59680
10720
95200
72880
4504, &c.

In the second of the foregoing Examples you may see the Generation of a compound Repetend: For after having used 7 Cyphers, I observed that if to the remainder 170 I had brought down another Cypher it would be 1700, which is the same I began with, consequently the same Figures, *viz.* 594405 would have repeated in the Quotient, and after them the same again, &c. Therefore I stop, and dash the first and last Figures as directed in Theorem the sixth.

In the first, third, and fourth of the last Examples, though I have carried the Decimal on to so many Places as is there wrought, I find they do not circulate, therefore I desist carrying them any farther; and here note, that whether they circulate or not, there is seldom a Necessity of getting more than 6 Places in the Decimal or Quotient, that being sufficiently exact for most Uses.

It may be observed that in the three last Examples, there doth not so many Figures arise, as there are Cyphers used in Division, therefore when I left off, I examined the Defect, and supply'd it with Cyphers

to the left Hand, before which I prefixed the mark of Distinction as before directed.

Secondly. To reduce the different Denominations of Money, Weights, Measures, &c. to their equivalent Decimal Expressions.

RULE.

PLACE the several Denominations given orderly under each other, the least being uppermost: Then reduce the least Denomination to the Decima Part of the next Superiour; to the right Hand of which superiour Denomination prefix the Decimal now found; and reduce this mixed number to the Decimal of the next Superiour; to the right Hand of which prefix the Decimal last found; and so proceed till you arrive at the Decimal of your intended Integer.

Note. ONE Denomination is reduced to the Decimal of the next Superiour by dividing the lesser Denomination by the number thereof contained in one of the next Superiour.

Reduce 10 *s.* 8 *d.*——13 *s.* 10½ *d.*——15 *s.* 9¾ *d.* and 19 *s.* 11¼ *d.* each to it's equivalent Decimal of a Pound Sterling.

12	8	4	2	4	3	4	1
20	10,6	12	10,5	12	9,75	12	11,25
	0,53	20	13,875	20	15,8125	20	19,9375
			0,69375		0,790625		0,996875

ANSWERS.

10 *s.* 8 *d.* is ,53 *l.*—13 *s.* 10½ *d.* is ,69375 *l.*—15 *s.* 9¾ *d.* is ,790625 *l.*—and 19 *s.* 11¼ *d.* is ,996875 *l.*

REDUCE

REDUCE 10 *oz.* 18 *dwts.* 16 *grs.* and 3 *qrs.* 16 ℔. 12 *oz.* each to their equivalent Decimals.

24 {4	16		16 {4	12
{6	(4		{4	(3
20	18,6		28 {4	16,75
12	10,93		{7	(41875
	0,91		4	3,598214, &c.
				0,899553, &c.

ANSWERS.

10 *oz.* 18 *dwts.* 16 *grs.* is ,91 of a ℔ Troy, and 3 *qrs.* 16 ℔ 12 *oz.* is ,899553 of a *C. wt.* Averdupoise.

REDUCE 10 *Inch.* 09 *Sec.* 07 *Parts*, a Duodecimal; and 48 *Min.* 37 *Sec.* 54 *Thirds*, a Sexagesimal; each to their equivalent Decimal.

12	7		60	54
12	9,583		60	37,9
12	10,79861		60	48,6316
	0,899884, &c.			0,810527

ANSWERS.

10 *Inch.* 09 *Sec.* 07 *Parts*, is ,899884, &c. of a Foot, and 48′ 37″ 54‴, is ,810527 of a Degree.

AFTER the ſame Manner may any other ſorts of contract Numbers be reduced.

BUT the Decimal Parts of a Pound Sterling may be expreſſed in one Line; thus: Write half the Number of Shillings in the Place of Primes, and reduce the Pence and Farthings into Farthings, which muſt poſſeſs the Places of Seconds and Thirds; and if the Shillings be odd, increaſe the Place of Seconds with 5: But increaſe the Place of Thirds with as many Units as there are times 24 in the Pence and Farthings.

LASTLY,

Lastly, If half the Number of Farthings in the Pence and Farthings [rejecting Sixpences] be divided by 12, the Quotient written after the three Places already found, shall compleat the Decimal.

Reduce 10 *s.* 8 *d.*—13 *s.* 10½ *d.*—15 *s.* 09¾ *d.*—19 *s.* 11¼ *d.*—1 *s.* 10¼ *d.*—8¾ *d.*—2½ *d.* and ¾ *d.* each to their equivalent Decimal of a Pound Sterling.

10 *s.* 8 *d.*	13 *s.* 10½ *d.*	15 *s.* 09¾ *d.*	19 *s.* 11¼ *d.*
l. ,533	*l.* ,69375	*l.* ,790625	*l.* ,996875
1 *s.* 10¼ *d.*	8¾ *d.*	2½ *d.*	¾ *d.*
l. ,0927083	*l.* ,0364583	*l.* ,010416	*l.* ,003125

By explaining one of these Examples, all the rest will be easy.

In the fifth, *viz*, 1 *s.* 10¼ *d.* First half of one is 0, write 0 in the Place of Primes; then 4 times 10 *d.* is 40 Farthings, and ¼ is 41, and because here is one 24, I add 1 to 41 and it makes 42; and because there is one Shilling, I increase the second Place, *viz.* 4 with 5, which now makes 92, this written after the 0, makes ,092. Lastly in 41 Farthings, rejecting 24 or Sixpence, there is 17 remains, half of which 8,5, which divided by 12, gives 7083 to be added to ,092. If this Process be well compared with the last Rule, the whole will be very plain.

Thirdly, To find the Value of a Decimal.

RULE.

Multiply the given Decimal by the Number of the next lesser Denomination, contained in one

one of that Denomination which the Decimal refpects as it's Integer; and from the Product cut off as many Places to the right Hand, as there are in the given Decimal; and this Decimal thus cut off, multiply by the next leffer Denomination, and from this Product cut off as before, and thus proceed till you are arrived at the leaft Denomination: Laftly, the feveral Denominations cut off on the left Hand are the Anfwers.

WHAT is the Value of ,72896 *l.*——,92384 *l.* and ,089235 *l.*

,72896	,92384	,089235
20	20	20
14,57920	18,47680	1,784700
12	12	12
6,95040	5,72160	9,416400
4	4	4
3,80160	2,88640	1,665600

ANSWERS. ,72896 *l.* is 14 *s.* 6¾ *d.*—,92384 *l.* is 18 *s.* 5½ *d.* and ,089235 *l.* is 1 *s.* 9¼.

THIS is fo plain it needs no Explanation.

WHAT is the Value of ,87628 of a ℔ Troy, and of ,798645 of a *C.wt.* Avoirdupoize?

,87628		,798645	
12	*Ounces.*	4	*Quarters.*
10,51536		3,194580	
20	*Penny weight.*	28	*Pound.*
10,30720		1556640	
24	*Grains.*	389160	
122880		5,448240	
61440		16	*Ounces.*
7,37280		7,171840	

ANSWERS

ANSWERS.

,87628 ℔ is 10 *oz.* 10 *dwts.* 7 *grs.* and ,798645 *C.wt.* is 3 *qrs.* 5 ℔ 7 *oz.*

But the Value of the Decimal of any part of a Pound Sterling may be expressed it one Line thus.

Double the Place of Primes for Shillings, and if the second Place be 5, or exceed 5, increase the double of the Primes by one for Shillings: Then the Figures in the second and third Places [rejecting 5 in the second Place] are so many Farthings, abating one for every 24, and the Remainder divided by 4 gives the Pence and Farthings.

What are the Values of ,92763 *l.* and ,87638 *l.* and ,09937 *l.* and ,0428 *l.* and ,0095 *l.*

,92763 *l.*	,87638 *l*	,09937 *l.*	,0428 *l.*
18 *s.* $6\frac{1}{2}$ *d.*	17 *s.* $6\frac{1}{4}$ *d.*	1 *s.* $11\frac{3}{4}$ *d.*	$10\frac{1}{4}$ *d.*

,0095 *l.*
$2\frac{1}{4}$ *d.*

SECTION III.

ADDITION and SUBTRACTION.

RULE.

ADD or subtract as in whole Numbers, and from the Sum or Difference cut off as many Decimal Places as are the greatest Number of Decimals

mals in any of the given Expressions: But observe that the separating Commas in each Expression be placed directly underneath each other; for then Unit, &c. (if any) will fall under Units, &c. and Primes, Seconds, Thirds, &c. under Primes, Seconds, Thirds, &c. for so they ought to do.

EXAMPLES in ADDITION.

347,256	3468,04973	8267,4031
5,61739	24,3675	8,965
,1725	148,952	453,720853
43,.	37,284695	76,98124
36,5	5,125	,2738107
432,54589	3683,778925	8807,3440037

EXAMPLES in SUBTRACTION.

From	384,76215	Minuend.	From	426,8
Take	86,2095.	Subtrahend.	Take	379,604832
	298,55265	Differ. or Rem.		45,195168

If Interminates are to be added, that have single Repetends.

RULE.

Make all the Repetends conterminous, that is, continue the repeating Figure till the Decimal Places in each are equal; observing that the repeating Figures be carried one Place farther to the right Hand than the greatest Number of Decimal Paces in any of the terminate Expressions: Then add as before; only increase the Sum of the right hand Row with as many Units as it contains Nines, and the Figure in the Sum under that Place will be a Repetend.

EXAMPLES.

$$\begin{array}{r}8439,6548\\281,04\dot{6}\\7042,3\dot{5}\\9,83\dot{7}\\\hline\end{array}\qquad\begin{array}{r}5391,357\\72,3\dot{8}\\187,2\dot{1}\\4,296\dot{5}\\\hline\end{array}\qquad\begin{array}{r}217,849\dot{6}\\42,17\dot{6}\\,52\dot{3}\\58,30048\\\hline\end{array}\qquad\begin{array}{r}876,29\dot{3}\\5,8764289\\,0358\dot{6}\\628,4593\dot{8}\\\hline\end{array}$$

In each of these Examples there are Repetends, and before they can be added, they must be made conterminous, and then they will stand as follows:

$$\begin{array}{r}8439,6548\\281,04\dot{6}66\\7042,3\dot{5}555\\9,83\dot{7}77\\\hline 15772,8948\dot{0}\\\hline\end{array}\qquad\begin{array}{r}5391,357\\72,3\dot{8}88\\187,2\dot{1}11\\4.296\dot{5}\\\hline 5655,253\dot{5}\\\hline\end{array}\qquad\begin{array}{r}217,849\dot{6}66\\42,17\dot{6}666\\,52\dot{3}333\\58,30048\\\hline 318,85014\dot{6}\\\hline\end{array}$$

$$\begin{array}{r}876,29\dot{3}3333\\5,8764289\\,0358\dot{6}666\\628,4593\dot{8}888\\\hline 1510,6650177\dot{8}\\\hline\end{array}$$

Here you may observe that in each Example the Repetends are carry'd one Place farther than the terminate Expressions, and to the Sum of that Row, there are as many Units added, as there were Nines in the Sum.

If Subtraction is to be performed with Interminates that have single Repetends.

RULE.

Make all conterminous as in Addition: But if the right hand Figure of the Subtrahend (being a Repetend) be bigger than the Figure over it in the Minuend

Minuend, instead of borrowing 10 as in Subtraction of whole Numbers or Terminates; borrow only 9 and this only in the right hand Figure, and this Figure of the Remainder will be a Repetend.

From	$476,3\cancel{2}$	$289,576$	$325,791\cancel{8}$	$643,9207$
Take	$84,769\cancel{7}$	$92,584\cancel{6}$	$37,095$	$583,7\cancel{6}$

THESE Examples being made conterminous as before directed, will stand thus:

$476,3\cancel{2}22$	$289,576\cancel{0}$	$325,791\cancel{8}$	$643,9207\cancel{0}$
$84,769\cancel{7}$	$92,584\cancel{6}$	$37,095\cancel{0}$	$583,7\cancel{6}666$
$391,552\cancel{4}$	$196,991\cancel{3}$	$288,696\cancel{8}$	$60,1540\cancel{3}$

SECTION IV.

MULTIPLICATION.

RULE.

PLACE and multiply as in whole Numbers, and from the Product towards the right Hand, cut off as many Places for Fractions, as there are fractional Parts in both Expressions or Factors together. But if it so happen, that there are not so many Places in the Product, supply the Defect with Cyphers to the left Hand.

3684,7928	,2365	,0347
84,216	,2435	,0236
221087568	11825	2082
36847928	7095	1041
73695856	9460	694
147391712	4730	,0081892
294783424	,05758775	
310318,5104448		

In the first Example there being 4 Decimal Places in the Multiplicand, and 3 in the Multiplier, which together are 7; I therefore cut off 7 Figures from the right Hand of the Product for Decimals; those to the left Hand being Integers.

In each of the second and third Examples the Decimal Places in both Factors are 8, but there turns out in the second Example only 7 Figures in the Product, and but 5 in the third Example's Product: Therefore in the second Example I annex 1 Cypher, and in the third 3 Cyphers, to supply the Defect.

In multiplying with single Repetends there are three Cases.

CASE I.

If the right hand Figure of the Multiplicand be a Repetend.

RULE.

Multiply the multiplicand as before, by every Figure in the Multiplier; observing to increase the right hand Figure of each resulting Line, by as many Units as there are Nines in the first Product of that Line; and the right hand Figure of each Line will be a Repetend; therefore in the adding the several Lines together, they must be made conterminous, as taught in Addition.

EXAM-

EXAMPLES.

First.

$$\begin{array}{r} 821,734\dot{8} \\ ,8 \\ \hline 657,3876\dot{4} \\ \hline \end{array}$$

Second.

$$\begin{array}{r} 3796,27\dot{8} \\ 4,\dot{6} \\ \hline 2277764\dot{0} \\ 1518509\dot{3}3 \\ \hline 1746,2857\dot{3} \\ \hline \end{array}$$

Third.

$$\begin{array}{r} 8946,8370\dot{6} \\ 48,57 \\ \hline 6262785594\dot{6} \\ 447341853\dot{3}3 \\ 7157469653\dot{3}3 \\ 357873482\dot{6}666 \\ \hline 434547,876328\dot{0} \\ \hline \end{array}$$

CASE II.

WHEN the Multiplier is a single Repetend.

RULE.

MULTIPLY by it as tho' it were a terminate Digit, setting the Product one Place forwarder than ordinary, towards the left Hand, and divide the Result by 9, continuing the Quotient (if needful) till you arrive at a Repetend; then beginning at the Place under the right hand Figure of the Multiplicand, cut off for Fractions as before, and you'll have the true Product.

EXAMPLES.

First.

438,6299
,6
9) 26317782
292,41980

Second.

5820,39462
,31
9) 4074276234
4526973593
1746118386
2198,8157453

Third.

47,63
2,845
9) 23815
26461
19052
38104
9526
135,53381

2,845
47,63
8536
170733
1991888
11382222
135,53381

THE third Example I have wrought by both Cases, where you may obſerve that they prove each other; the Reſult turning out exactly the ſame.

IN this ſecond Caſe, if there are any other Figures in the Multiplier, beſide the Repetend, multiply by them like terminate Digits.

CASE III.

WHEN the Multiplicand and Multiplier are each a ſingle Repetend.

RULE.

IN multiplying the Multiplicand by each Figure in the Multiplier, obſerve the Direction given in CASE I. but the firſt Line (or that Line produced by multiplying the Multiplicand by the Repetend in the

the Multiplier) must be managed as directed in Case II.

EXAMPLES.

```
   463,9704                  862357,97
     8,641                       35,16
9) 927940                9) 517414758
   1031045432, &c.         57490528148
  18558817                 86235797
 278382266               4311789611
37111763855              25870737666
4009,73568442, &c.       30326253,59814
```

```
     ,5392765
      953,7
9) 37749358
    41943732098, &c.
   16178296
  269638277
 48534890
514,350094765432098
```

By dividing by 9 in each of these Examples there results a compound Repetend, now having finished the Multiplication, I make all the Lines conterminous, but no farther than the first Figure of the compound Repetend, then having added the several Lines together as before directed, I dash the right hand Figure of the Product, for the first Figure of a compound Repetend; the rest of which I set orderly after as they follow in the given compound Repetend.

When there happens many Decimal Places in each Factor, there doth usually arise many useless Figures

Figures in the Product, to avoid which observe the following

RULE.

Having determined how many Decimal Places are to be had in the Product, under THAT Place in the Multiplicand set the Unit's Place of the Multiplier, and invert (or turn round) the Order of all the other Places of it: Then in multiplying, begin at that Figure of the Multiplicand which stands over the Figure you are then multiplying with; setting down the first Figure of every Product directly underneath one another, observing to have a due regard in carrying the Increase which would arise out of the Figures to the right Hand of that Figure in the Multiplicand you then begin with, and in the Product the lowest Place is uncertain.

But if from the Figures omitted you carry'd 1 from 5 to 15; 2 from 15 to 25; 3 from 25 to 35; &c. instead of carrying 1 for every 10, &c. the right hand Place in the Product would be generally exact.

EXAMPLES.

Multiply 384,672158 by 36,8345.

Now seeing there would be 10 Decimal Places in the Product, whereof the greatest Part are unnecessary; therefore I intend to keep only 4 Decimal Places in the Product.

384,672158	Multiplicand.	384,672158
5438,63	Multiplier inverted.	36,8345
115401647 .		1923 360790
23080329 . .		15386 88632
3077377 . . .		115401 6474
115402		3077377 264
15387		23080329 48
1923		115401647 4
14169,2065		14169,2066 038510

Here I have wrought the Example both ways, by which may be easily seen what is saved by the contracted way.

In this Example because I intended to keep 4 Decimal Places in the Product, I set 6, the Unit's Place of the Multiplier under 1, the *4th* Place in Decimals of the Multiplicand, and invert the Order of all the rest of the Figures: Then I say 3 times 8 is 24, and carry 2; 3 times 5 is 15, and 2 is 17, now set down the 7 and carry 1, *&c.* because this is the Product arising by Multiplying the 5 that stands over the 3.

Again, 6 times 8 is 48, and carry 4; 6 times 5 is 30, and 4 is 34, and carry 3; 6 times 1 is 6, and 3 is 9. Now being come to the Figure over the 6, I set down 9, *&c.*

Again, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17; now being come to the Figure over the 8, I set down 7, and carry 1, *&c.* Proceeding in like Manner with every Figure in the inverted Multiplier, till all is done.

Mul-

MULTIPLY 3,141592 by 52,7438, and to reserve 4 Decimal Places in the Product.

3,141592
8347,25
1570796
62831
21991
1257
94
25
165,6994

MULTIPLY 257,356 by 76,48, and to have the Product only in whole Numbers.

257,356		257,356
84,67		7648
18015	20	58848
1544	102	9424
103	1544	136
20	18014	92
19682	19682,	58688

SECTION V.

DIVISION.

RULE.

DIVIDE as tho' all were Integers; but so place the separating Comma in the Quotient, that the Sum of the decimal Places in the Divisor and

and Quotient may be equal to those in the Dividend———But if the Decimal Places in the Divisor be more than those in the Dividend add Cyphers as Decimals to the Dividend, till the Number of Decimal Places in the Dividend is at least equal to those in the Divisor.

But if there arise not so many Places in the Quotient as the Rule requires, supply the Defect with Cyphers to the left Hand.

```
43,6) 3424,6056 (78,546
       3726
        2380
         2005
          2616

,675) 3877875,000 (5745000
      5028
       3037
        3375
           000

,347) ,0081892 (,0236
         1249
          2082
```

If these Examples be compared with the foregoing Rule, it will be very easy to see how they are performed, and the Quotient rightly adjusted.

If the Dividend be a Repetend, bring down the repeating Figure to the Remainders in order to have your Quotient to it's intended Exactness.

4,72) 8,73 (1,76553, &c.
3613
3093
2613
2533
1733
317, &c.

54,283) ,876527 (,016147, &c.
333697
79997
217147
400157
20176, &c.

If the Divisor be a Repetend——Then imagine the separating Point of the Divisor moved as many Places to the left Hand as the Divisor's Repetend consists of Places, and then subtract it from the Divisor itself: Also remove the separating Point of the Dividend towards the left Hand as many Places as the Divisor's was; and this subtract from the Dividend itself: Then divide with those Remainders as in the foregoing Directions, and the Result is the Quotient sought after.

EXAMPLES

EXAMPLES.

First. 8946,83706) 434547,876328 (48,57
894,68370 43454,7876328
8052,15336) 391093,0886952
6900695429
4589727415
5636507352
0000000000

It will be easy to see how this Example is performed, by observing the Rule; and by comparing it with Example 3*d* to Case I. of Repetends in Multiplication, you will see how these two Rules prove each other.

Second. 748,64) 47,464057 (,0634
74,86 4,746405
673,78) 42,717652
229085
269512
0

Third. 2,848) 135,53381 (47,63
284 13,55338
2,561) 121,98043
19540
16134
7683
0

Fourth. 76,47) 82937643 (108,44
764 8293764
68,83) 74643879
58138
30747
32159
4627

DIVISION like Multiplication may be contracted as follows:

HAVING determined of what Value the first Figure in the Quotient will be: Let each Remainder be a new Dividend, and for each such new Dividend prick off one Figure from the right Hand of the Divisor: Observing at each Multiplication to have regard to the Increase of the Figures so cut off; and the last Figure in the Quotient is generally uncertain.

EXAMPLES.

384,672158) 14169,2066239510 (36,8345
..... 11540 16474
262904188 .
230803294 .
32100894 . .
30773772 . .
1327122 . . .
1154016 . . .
173106 . . . .
153868 . . . .
19238 . . . . .
19233 . . . . .

9,365407
......

```
9,365407) 87,076326 (9297655
 ......   84 288663
          ---------
           2787663 .
           1873081 .
           -------
            914582 ..
            842886 ..
            ------
             71696 ...
             65557 ...
             -----
              6139 ....
              5619 ....
              ----
               520 .....
               468 .....
               ---
                52 ......
                46 ......
                --
                 6 ......
```

THIS will not be difficult if it be carefully examined.

SECTION VI.

Of COMPARISON *or* PROPORTION.

DEFINITIONS.

I. THE comparing things of a like Kind to one another may be considered two Ways:

First, BY how much one thing exceeds, or is greater than another, and this is called *Difference*.

Secondly, WHAT Part or Parts one thing is of another, and this is called *Ratio*. And two or more Ratio's make a Proportion, *viz.*

When one Number is to be divided by another; then the dividing Number is ſaid to be in the ſame Proportion to the Number to be divided; as Unity or one is in proportion to the Quotient; that is,

The dividing Number is as often contained in the Number to be divided: As Unity or 1 is contained in the Quotient. Therefore Ratio (and two Ratios make Proportion) is nothing more than how often one Number is contained in another; that is, it is the Quotient of one Number divided by antoher.

II. There cannot be leſs than two Numbers or Terms in any Proportion: The firſt of which, or the Term by which the Compariſon is made, is called the *Antecedent*: And the Second; or the Term to which the firſt is compared, is called the *Conſequent*.

III. When two Differences are equal, the Numbers or Terms by which theſe Differences were made, are ſaid to be in Arithmetical Proportion or rather Progreſſion. Thus 4, 6, 8, 10, are Numbers in Arithmetical Proportion, becauſe the Difference between every one of the Terms are equal; and the like in other Numbers.

But when two Ratios (*i. e.* Quotients) are equal, the Numbers or Terms by which theſe Ratios were made, are ſaid to be in Geometrical Proportion. Thus 3, 12, 4, 16, are Numbers in Geometrical Proportion, becauſe twelve divided by 3, gives 4; and 16 divided by 4 gives 4 alſo; that is, they give equal Quotients; therefore the Numbers are in Geometric Proportion; and the ſame is to be underſtood of other Numbers that will give equal Quotients.

IV. When of several Terms or Numbers, the Quotient of the first and second is the same with that of the second and third, and the same with the third and fourth, &c. those Numbers are said to be in continued Geometric Proportion: Thus, 4, 12, 36, 108, &c. are Numbers in continued Geometric Proportion, for the Quotient of any two adjacent Terms is 3.

THEOREMS.

I. If there are three Terms in Geometrical Proportion, the first Term multiplied by the third; gives a Product equal to the second Term multiplied by itself; that is, the second Term squared.

Hence the second Term is called *a mean Proportional between the Extreams:* viz. *the first and third Terms.*

II. If there are four Terms in Geometrical Proportion; the Product of the two extream Terms is equal to the Product of the two mean Terms.

Hence the second and third Terms are called *mean Proportionals between the first and fourth Terms.*

Now it is evident, that if the Product of the second and third Terms be divided by the first Term, the Quotient will be the fourth Term.

And hence ariseth the Method of operating the Rule of three.

The Rule of three is so called, because three Numbers or Terms are given to find a fourth: Thus, if the Numbers 3, 12, and 18 were given to find a fourth Proportional.

Now multiplying the second Term by the third, or (which is all one) the third by the second, *viz.* 18 by 12, the Product will be 216; this 216 divided by the first Term 3, gives 72 in the Quotient, for the fourth Term.

Now by comparing of the Terms together, you will find that the ſecond Term 12 as often contains the firſt Term 3; as the fourth Term 72, contains the third Term 18: Alſo that the third Term 18 as often contains the firſt Term 3, as the fourth Term 72 contains the ſecond Term 12; that is, the Ratio or Proportion of 3 to 12 is the ſame as the Ratio of 18 to 72. Alſo the Ratio of 3 to 18 is the ſame as the Ratio of 12 to 72; and the like in any other Numbers. And this is the Doctrine of Proportions.

The Rule of three being moſtly concerned in finding a fourth Number proportional to three Numbers or Terms given, differing in Signification and Denomination, the greateſt Difficulty lies in ſtating the Terms; that is, in placing them in proper Order to be multiplied and divided according to the foregoing Directions; to do which obſerve the following Directions.

Consider what Sort or Quality the Term required is of: And of the three Terms given, ſet that which is of the ſame Quality with the Term required, for the ſecond Term; and the remaining two Terms for Diſtinction ſake, call the *Antecedential Terms*.

Then by the Tenour of the Queſtion, you'll eaſily be able to judge whether the fourth Term, or Term required, will be leſs or greater than the ſecond Term.

Having conſidered that, place the Antecedential Terms for firſt and third in the ſame Order; that is, if the ſecond term be leſs than what the fourth will be, place the leaſt of the Antecedential Terms for the firſt Term, and the other for the third: But if the ſecond Term is greater than what the fourth will be; place the greater of the Antecedential

dential Terms for the firſt Term, and the other for the third: And then the three Terms are ſtated fit for Solution,

THEN if any, or all of the Terms be of different Denominations, reduce the leſſer Denominations to the decimal Part or Parts of the greater Denomination; as taught in SECT. II. obſerving that the leſſer Denominations in the firſt and third Terms be reduced to the decimal Part or Parts of one and the ſame Denomination.

THEN having multiplied the ſecond and third Terms, together by the Rules in SECT IV. and divided the Product by the firſt Term, and adjuſted the Quotient by the Rules given in SECT. V, you may then call the Quotient the *fourth Term*, or *the Thing ſought after*; and may proceed to value it by the Rules given in SECT. II.

QUESTION I.

WHAT will $326\frac{1}{4}$ ℔ of Tobacco come to, at 3 *s.* 6 *d.* for $1\frac{1}{2}$ ℔?

HERE the Quality of the fourth Term is Money, *viz.* the worth of $326\frac{1}{4}$ ℔ of Tobacco: And among the three Terms given, one is Money, *viz.* 3 *s.* 6 *d.* the worth of $1\frac{1}{2}$ ℔.

Now I can very eaſily ſee that $326\frac{1}{4}$ ℔ will come to more than $1\frac{1}{2}$ ℔, therefore the fourth Term will be greater than the ſecond; and the Terms ſtated will ſtand thus: If $1\frac{1}{2}$ ℔ coſt 3 *s.* 6 *d.* what will $326\frac{1}{4}$ ℔ come to; and the Terms reduced into Decimals will ſtand thus:

IF 1,5 ℔ —— ,175 *l.* —— 326,25 ℔

,175

163125

554625

1,5) 57,09375 (38,0625 *l.*

120

93

37

75

HERE the third Term being multiplied by the second; and the Product being divided by the first Term gives 38,0625 *l.* in the Quotient; which being valued as directed in SECT. II. gives 38 *l.* 1 *s.* 3 *d.* the Money that 326¼ ℔ of Tobacco will come to, at the Rate of 3 *s.* 6 *d.* for 1½ ℔.

QUESTION II.

WHAT is the worth of 19 *oz.* 3 *dwts.* 5 *grs.* of Gold, at 2 *l.* 19 *s.* 4 *d. per Ounce?*

IF 1 *oz.* — 2 *l.* 19 *s.* 4 *d.* — 19 *oz.* 3 *dwts.* 5 *grs.*

24	4	5
	6	(125
20		3,2083
		0,160416

Or thus in Decimals:

IF 1 *oz.* —— 2,96 *l.* —— 19,160416 *oz.*

2,96

9) 114962500

127736111

1724437500

3832083333

56,8425694

BECAUSE

BECAUSE the first Term here is Unity or 1, which neither multiplies nor divides, therefore the Answer is produced by multiplying the second and third Terms together; and the Product being valued as taught in SECT. II. gives 56 *l.* 16 *s.* 10$\frac{1}{4}$ *d.* the Value of the Gold, as sought after.

QUESTION III.

WHAT is the Worth of 827$\frac{3}{4}$ *Yards* of Painting at 10$\frac{1}{2}$ *d. per Yard.*

IF 1 *yd.* — ,04375 *l.* — 827,75ø0 *yd.*

```
         57 340,0
         --------
         33 1100
        . 2 4832
            5794 } ANS. 36 l.
             414 } 4 s. 3½ d.
         --------
         36,2140
```

Now because there would be 7 decimal Places in the Answer, whereof 4 are more than sufficient, I therefore in order to get 4 Decimals in the Product, set the Place of 0 Units under the fourth Decimal in the Multiplicand, and invert the Order of the rest as directed in SECT. IV.

QUESTION IV.

LENT my Friend 34 *l.* for $\frac{5}{8}$ of a Year, how much ought he to lend me $\frac{9}{12}$ of a Year, to requite my Kindness?

,75 *y.* — 34 *l.* — ,625 *y.*

```
             34
           ----
           2500
          1875
          -----
    75) 21,25 (28,333   { ANSWER.
        -----           { 28 l. 6 s. 8 d.
         625
          250
          ---
           25
```

QUESTION V.

IF $\frac{3}{4}$ of a Yard of Cloth, that is $2\frac{1}{4}$ Yards broad, make a Garment, how much of another Sort, that is but $\frac{4}{5}$ of a Yard wide, will make the same Garment?

,8 *y. b.* — ,75 *y. l.* — 2,25 *y. b.*

```
                ,75
               ----
               1125
              1575
              ------
     ,8) 1,6875 (2,1093, &c.
```

ANSWER. 2 Yards and 1 Nail.

QUESTION VI.

IF when the Bushel of Wheat cost 4 *s.* 9 *d.* the Penny Loaf weigh'd $10\frac{2}{3}$ Ounces, what will it weigh when the Bushel of Wheat is sold for 8 *s.* 10 *d.*

d.	*oz.*	*d.*	
,4416 ——	10,6 ——	,2375	
		10,6	
	9)	14250	,7
		15833	12
		23750	8,4
	,4416)	2,53333(5,7	
	416	25333	ANSWER,
	,4000	2,28000	$5\frac{2}{3}$ *oz.*

FOR the ſpeedy operating thoſe Examples in the Rule of Three, where the firſt Term is Unity or 1; there are ſeveral compendious Rules given in the Books of Arithmetic, and by the Authors called *Practice* (which is nothing more than the Application of the Doctrine of Aliquot Parts) and theſe Rules are nearly as many as there have been Writers of Arithmetic: Therefore, as it would be an endleſs Task, to give all the eaſy Methods of Operation adapted to particular Caſes; I ſhall only mention ſome general Directions, whereby with the Judgment of the Operator, the moſt common Caſes may be expeditiouſly ſolved. And,

Firſt. AN Aliquot Part is ſuch a Part of ſome whole, that the Aliquot Part taken a certain Number of Times will make the whole: Thus three is an Aliquot Part of 18, becauſe 3 taken 6 times will make 18.

THE

THE following Tables ſhews the Aliquot Parts of a Pound, and of a Shilling.

10 *s.* is $\frac{1}{2}$	of a Pound.	6 *d.* is $\frac{1}{2}$	of a Shilling.	and $\frac{1}{40}$ of a Pound.
6 *s.* 8 *d.* - $\frac{1}{3}$		4 *d.* - $\frac{1}{3}$		and $\frac{1}{60}$
5 *s.* - - - - - $\frac{1}{4}$		3 *d.* - $\frac{1}{4}$		and $\frac{1}{80}$
4 *s.* - - - - - $\frac{1}{5}$		2 *d.* - $\frac{1}{6}$		and $\frac{1}{120}$
3 *s.* 4 *d.* - $\frac{1}{6}$		1 $\frac{1}{2}$ *d.* - $\frac{1}{8}$		
2 *s.* 6 *d.* - $\frac{1}{8}$		$\frac{3}{4}$ *d.* - $\frac{1}{16}$		
2 *s.* - - - - - $\frac{1}{10}$				

IN all Caſes where the Value of ſome given Things are required.

TAKE the moſt convenient Aliquot Parts of the greater Denomination of the Things given to be valued, for the ſeveral inferior Denominations of the Price: And for the Value of the inferior Denominations of the Things given to be valued, take the moſt convenient Aliquot Parts of the Price.

SECT.

SECTION VII.

EXTRACTION *of the* SQUARE ROOT.

EXTRACTION of the ſquare Root is the finding ſuch a Number out of any Number given, as being multiplied into itſelf will either produce the given Number, or come very near it.

THEREFORE a ſquare Number is that which is produced by the Multiplication of two equal Numbers together: Thus 9 is a ſquare Number, as being produced by multiplying 3 by 3. Alſo 144 is a ſquare Number, equal to 12 times 12.

THE following Table exhibits the Roots and Squares of all ſingle Numbers.

ROOTS.	1,	2,	3,	4,	5,	6,	7,	8,	9.
SQUARES.	1,	4,	9,	16,	25,	36,	49,	64,	81.

WHEN any Number is given to extract the ſquare Root out of it, the firſt thing muſt be to point it into Periods of two Figures each; that is, always beginning at the left hand Figure, or Figure that ſtands in the Place of Units, over which put a Point, and point every ſecond Figure counting from the right Hand towards the Left;

Left; but in pointing of Decimals, point every ſecond Figure from the left Hand towards the Right.

Then ſeek the greateſt Square contained in the firſt Period towards the left Hand, whoſe Root place in the Quotient as in Diviſion, and the Square of the Root ſet under the Period out of which it was taken; from which Period ſubtract it, and to the Remainder bring down the next Period, which call a *Reſolvend*.

Then double the Root or quotient Figure for a Diviſor, and ſeek how often it may be had in all the Figures of the Reſolvend, excepting the right hand Figure and the Figure reſulting, place both in the Quotient, and on the right Hand of the Diviſor; then multiply the increaſed Diviſor by the Figure laſt put in the Quotient, and the Product place under the new Reſolvend, (obſerving to put Units under Units) from which ſubtract it, and to the Remainder bring down the next Period for a new Reſolvend; and double all the Quotient for a Diviſor; and proceed as before till all the Periods are brought down.

If at laſt there happen to be a great Remainder, and you are willing to have the Root more accurate, by increaſing it with a decimal Fraction. Then to the Remainder annex two Cyphers, and proſecute the Work as before, &c. always adding two Cyphers to the Remainder, till you arrive at your deſired Exactneſs.

EXAMPLE I.

What is the ſquare Root of 132496.

13249

```
     132496(364
       9
  66) 424
      396
 724) 2896
      2896
```

THE first Period towards the left Hand is 13, the greatest Square in which is 9, whose Root 3, I place in the Quote, and the Square 9 I place under the 13, and subtracting, there remains 4, to which I bring down the next Point 24, which makes the Resolvend 424; to the left Hand of which I draw a curv'd Line, and at some Distance therefrom I put the double of 3, *viz.* 6, and inquire how oft this 6 may be had in 42, and I find 6 times, which 6 I place in the Quote, and also on the right Hand of the Divisor 6, and the increased Divisor is 66, which I multiply by the 6 in the Quote, and it gives 396, which subtracted from 424 leaves 28, to which the next Period 96 is brought down; and 36, the Quotient, doubled makes 72, the Divisor, which in 289 goes 4 times, which 4 placed in the Root and Divisor, and the new Divisor 724 multiplied by 4, gives 2896, so that nothing remains: Therefore I conclude 364 to be the true Root; for 364 multiplied by 364 gives 132496 the first Number given.

EXAMPLE II.

WHAT is the Square Root of 763958207163?

```
                 763958207163 (874047,027
                 64
         167)    1239
                 1169
          1744)  7058
                 6976
        174804)  822071
                 699216
      1748087)   12285563
                 12236609
    174809402)   489540000
                 349618804
   1748094047)   13992119600
                 12236658329
                  1755461281, &c.
```

In this Example after having brought down all the Figures in the given Number, to the Remainder I bring down Cyphers, and proceed with the Work in the ſame Manner as if they had been Numbers given; which Work I have continued till having got three Places in Decimals in the Root, I deſiſt, as thinking that ſufficient: And here obſerve, that if after having extracted the Root of any given Number, there be a Remainder, then, tho' you continue the Work on ever ſo far, there will ever be a Remainder. For no Number whatever [whoſe right hand Figure is one of the 9 Digits] will produce by being ſquared, a Number, whoſe right hand Place will be a Cypher.

You may always determine as ſoon as you have pointed the given Number, how many Figures you will have in whole Numbers in the Root; for the Places of whole Numbers in the Root, is always equal

equal to the Number of Points in the given whole Numbers, and the ſame for Decimals.

More EXAMPLES.

WHAT is the ſquare Root of

36372961	Answer.	6031
64681024		4968
1,0609		1,03
911236798,794365		30186,699

THESE Examples are operated in the ſame Manner as the preceding ones.

THERE are many Uſes to which the ſquare Root may be applied, but in this Place I ſhall only mention one, *viz.*

TWO Numbers being given, between them to find a mean Proportional.

RULE.

MULTIPLY the two Numbers together, and out of the Product extract the ſquare Root, which Root is the mean Proportional required.

EXAMPLE I.

WHAT is the mean Proportional between 3 and 12; 3 multiplied by 12, is 36, whoſe ſquare Root is 6, the Mean required: For as 3 is to 6, ſo is 6 to 12, by the Rules of Proportion.

EXAMPLE II.

FIND a mean Proportional between 4276 and 842.

```
    4276                3600392 (1897,4, &c.
     842                1
   -----
    8552             28) 260
   17104                224
  34208
  -------           369) 3603
  3600392                3321

                    3787) 28292
                          26509

                 3794,4)178300
                        151776

                         26524, &c.
```

So 1897,4, *&c.* is the mean Proportional required.

SECTION VIII.

EXTRACTION *of the* CUBE ROOT.

IF the finding such a Number out of a Number given, as being multiplied by itself, and this Product multiplied by the same Number, shall produce a Product either equal, or nearly so, to the Number given.

THEREFORE a Cube Number is produced by multiplying three equal Numbers together. Thus 27 is a Cube Number, produced by multiplying 3 by

by 3, and the Product 9 by 3 again. Alſo 1728 is a Cube Number produced by multiplying 12 by 12, and the Product 144 by 12 again.

THE following Table exhibits the Roots and Cubes of the ſingle Numbers.

ROOTS.	1,	2,	3,	4,	5,	6,	7,	8,	9.
CUBE.	1,	4,	27,	64,	125,	216,	343,	512,	729.

WHEN any Number is to have the Cube Root extracted, firſt point it into Periods of three Figures each, beginning at Units, and as many Points as there are, ſo many Figures muſt be in the Root.

THEN ſeek the greateſt Cube in the firſt Period to the left Hand, whoſe Root place in the Quotient, and the Cube of this Root under the Period out of which it was taken, from the Period ſubtract the Cube, and to the Remainder bring down the next Period and call this the Reſolvend, under which draw a Line; then triple the Square of the Root, that is, multiply it by three, and this triple Square ſet under the Reſolvend, ſo that the triple Square's Units may ſtand under the Reſolvend's Hundreds; then triple the Root, and ſet it ſo, that it's Tens may ſtand under the triple Square's Units: Add the triple Square and triple Root together, and their Sum call a *Diviſor*, and inquire how often it may be had in the Reſolvend [excepting it's right hand Figure], and the Number reſulting place in the Quotient, and under the Diviſor draw a Line.

THEN multiply the triple Square by the laſt Figure placed in the Quotient, ſetting the Units of the Product under the Units of the triple Square: Alſo, multiply the triple Root by the Square of the laſt quotient Figure, ſetting the Units of the Product

duct under the Units of the triple Root: Then set the Cube of the last Quotient Figure so, that Tens in the said Cube may stand under Units in the foregoing Line: Add these three last Lines together, and their Sum subtract from the Resolvend, to the Remainder bring down the next Period, if any, for a new Resolvend; and proceed for a Divisor, &c. as before.

EXAMPLE I.

WHAT is the Cube Root of 48228544?

	48228544	(364
	27	
	21228	Resolvend.
add	27	Triple Square of 3.
	09	Triple of 3.
	279	Divisor.
add	162	Triple Square of 3 multiplied by 6.
	324	Triple of 3 multipl. by Square of 6.
	216	Cube of 6.
	19656	Subtrahend.
	1572544	Resolvend.
add	3888	Triple Square of 36.
	108	Triple of 36.
	38988	Divisor.
add	15552	Triple Square of 36 mult. by 4.
	1728	Trip. of 36 mult. by Sqr. of 4.
	64	Cube of 4 Subtracted.
	1572544	

If the Operation of this Example be well considered, and compared with the foregoing Rule, it will be easy to conceive how any other Example of the like Nature may be wrought ; and here observe that when the Cube Root is extracted to more than two Places, there is a Necessity of doing some Work upon a spare piece of Paper, in Order to come at the Root's triple Square, and the Product of the triple Root by the Square of the quotient Figure, &c.

In this Example the given Number is a Cube Number, and therefore at the End of the Operation there remained nothing; for 364 multiplied by by 364, and the Product multiplied by 364 gives 48228544, the given Number.

But if the Number given be not a Cube Number, then to the last Remainder always bring down three Cyphers, and work anew for a decimal Fraction if needful.

More EXAMPLES.

What is the Cube Root of

389017	Answers.	73
4092727		103
27054036008		3002
219365327791		6031
122615327232		4968

These Examples are all operated in the same Manner as the foregoing one.

There are many Uses of the Cube Root, but I shall only mention one here, *viz.* To find two mean Proportionals between two given Numbers.

RULE

RULE.

DIVIDE the greater Extream by the leſſer, and the Cube Root of the Quotient multiply by the leſſer Extream, and the Product is the leſſer Mean. Multiply the ſaid Cube Root by the leſſer Mean, and the Product is the greater mean Proportional.

Note. THIS is only underſtood of thoſe Numbers that are in continued Geometric Proportion.

EXAMPLE I.

WHAT are the two mean Proportionals between 4 and 108?

108 divided by 4 gives 27, whoſe Cube Root is 3; and the leſſer Extream 4, multiply'd thereby, gives 12 for the leſſer Mean; and 12 multiplied by the ſaid Root 3, gives 36 for the greater Mean.

FOR as 4 is to 12, ſo is 36 to 108.

EXAMPLE II.

FIND the two Geometric Means between 8 and 1728?

8) 1728 (216, whoſe Cube Root is 6. And 6 times 8 is 48, the leſſer Mean, and 6 times 48 is 288 the greater Mean.

FOR as 8 is to 48, ſo is 288 to 1728.

WHEN any given Number is multiplied into itſelf any Number of times, the Product is called a *Power*; and is named according to the Number of Times the given Number, or Root, is ſo multiplied.

THUS

Thus 3 multiplied by 3 gives 9, which is the *Square*, or second Power of 3; and 3 by 3, and this Product by 3 gives 27, which is the *Cube*, or third Power of 3; and 3 by 3, and this Product by 3, and this Product by 3, gives 81, which is the *Biquadrat*, or fourth Power of 3; and 3 by 3, and this Product by 3, and this Product by 3, and this Product by 3, gives 243, which is the *Surfolid*, or fifth Power of 3.——And the like of any other Number given: And this sort of Multiplication of any given Number is called *Involution*.

Note. The Power any Number is raised to, is called it's *Dimension*, and the Number given to be raised is called the *Root* or *Side*, or *First Power*, or *First Dimension*.

EXAMPLE.

Suppose it was required to involve the Number 6 four times; that is, to raise it to the fourth Power, or fourth Dimension?

Then 6 by 6 is 36, the *Square* or second Power; and 36 by 6 is 216, the *Cube* or third Power; and 216 by 6 is 1296, the *Biquadrat* or fourth Power.

There are particular Rules for extracting the Root out of a Number of any given Dimensions; but no Extraction farther than the Cube Root being necessary in Mensuration, I have omitted all above, and for which refer the more inquisitive Reader to *Ward's young Mathematicians Guide*, where he will find enough of that Business well treated off. But as the foregoing common Rule which is given in other Books of Mensuration, is very tedious and operose: I shall before I conclude this Section give a Rule for extracting the Cube Root taken from a Series of the aforesaid *Ward's* Book; which is thus:

Point

POINT the given Number, and seek the greatest Cube in the first Period to the left Hand, whose Root place as a Quotient as before directed; and subtract the foresaid Cube from the foresaid Period, and to the Remainder bring down all the remaining Figures in the given Number; then to the Root now found annex as many Cyphers as there are remaining Points over the given Number; and this Result multiply by 3, (the Cube's Dimension) by which Product divide the Difference between the given Number, and the aforesaid Cube; and the Quotient point anew as in the Square; that is, begin at the Place of Units, and point every other Figure to the left and right [observing that the Number of Points over the Integers of this Quotient be no more than the Points brought down to the Difference between the first Period and the aforesaid Cube]. Then make the Root first found a Divisor, and inquire how often it may be had in the first Period of this Quotient, [excepting the Figure under the Point], and the Figure resulting place as a Quotient, and also on the right Hand of the Divisor; then multiply this increased Divisor by the quotient Figure, and the Product subtract from the Period out of which this last quotient Figure was taken, and to the Remainder bring down the next Period (if any) and this divide by the last increased Divisor, *&c.*

AN Example or two will make this Rule very intelligible.

EXAMPLE I.

WHAT is the Cube Root of 12812904?

28211,

```
         ˙  ˙  ˙
        12812904  (200
         8           3
  6,00)*4812904    600
     23) 8021,5 &c.   (34
         69            200
    234) 1121          234
          936
          185
```

I First begin at 4 the Place of Units, over which I put a Point, and omitting two Figures, I put another Point over the 2, and omitting two Figures more, I put another Point over the left hand Figure 2. Now here are 3 Points, and therefore I know I must have three Places of Integers in the Root. Then beginning with the first Period 12, I find the greatest Cube therein is 8, whose Root is 2; so I place 8 under 12, and 2 as a Quotient. Then I subtract 8 from 12, and to the Remainder 4 I bring down the remaining Figures (which occupy the Places of two Points or Periods.) Then to the Quotient (2) I annex two Cyphers, for the two Points remaining over the given Number [for the Quotient 2 is in Reality 200, because it possesses the left hand Place (*i. e.* the highest) of the three Integers, which is the Place of hundreds,] and this 200 I multiply by 3 (the Cube's Dimension) and the Product 600 I make a Divisor, by which I divide 4812904, which is the Difference between 8 the Cube of 2, and the given Number, and the Quotient is 8021,5, &c. Then I begin at 1, the Place of Units, and point as directed in the Square, till I have put as many Points as I annexed Cyphers

to the firſt Root, which in this Example is two. Then to the right and left Hands of this Number *viz.* 8021, *&c.* I draw curved Lines, as in Diviſion, and make the firſt Root 2 a Diviſor, inquiring how oft it may be found in the firſt Period 80 [excepting the Place under the Point, that is, ſay how often 2 in 8,] and it gives 3, which I place in the Quotient, and alſo, on the right Hand of the Diviſor 2, which now becomes 23. This 23 I multiply by the Quotient 3, and the Product 69 I ſubtract from 80, and to the Remainder 11, I bring down the next Period 21, which makes 1121. Now 23 is the Diviſor, which in 112 (omitting the right hand Place 1) goes 4, which 4 I place in the Quotient, and on the right Hand of 23, which now becomes 234, this 234 multiplied by the Quotient 4, gives 936, and ſubtracting, there remains 187.

Now this Quotient 34 added to the firſt Root 200, makes 234, and if this 234 be cubed, it will be 12812904, which was the Number firſt given, and therefore I conclude 234 to be the true Root required.

EXAMPLE II.

Extract the Cube Root out of 92398647506217, ſo that the Root may conſiſt of eight Figures or Places.

```
            9239864750621 7   (40000
            64                     3
            ------------------  ------
12,0000)    2839864750.6217   120000
            -----------------
   45)      236655395          (52
            225                400, &c.
            -----              -----
  452)      1165               452, &c.
            904
            ----
```

THEN 452 cubed is 92345408. And

```
      9239864750621 7
      92345408 subtract
      -----------------
      5323950 6217
```

```
             9239864750621 7      (45200
             92345408                  3
             ------------------   ------
                                  135600
1356,00)     532395062,17  (39621,727293
             -------------
45208)       392621,727293 (08,684, &c.
             361664
             ------
452086)      3095772          45200
             2712516             08,684
             -------          ---------
4520868)     38325672         45208,684
             36166944         The Root.
             --------
45208684)    215872893
             180834736
             ---------
             35038157, &c.
```

IN this Example having found 3 Places in the Root as before directed, I desist finding more till I have made a new Involution. Therefore I cube the Root 452, which gives 92345408, which used as the first Cube of 4 was, *viz.* subtracted from

the given Number 92398647506217, leaves 53239506217, which divided by the Root 452 with two Cyphers annexed, (for the remaining two Points over the Number given,) *viz.* 45200, gives 392621,727293, which pointed as in the Square, and the Root 452 uſed as a Diviſor, &c. as before directed, gives 08,684, which added to the foregoing Root 45200, gives 45208,684 for the Root required.

It is found by Experience, that this Method very ſeldom fails giving the Root of any Number true to three Places at leaſt, at the firſt operation; and if the ſecond Place in the Root is poſſeſſed by a Cypher, or an Unit, you may get four or five Places true at the firſt Operation frequently.

But if you make a ſecond Operation, as in the foregoing Example, you may depend on having the Root true to nine Places; but if more Accuracy be required, and you make a third Operation, you will have the Root true to 27 Places, each Operation tripling the Figures found in the laſt Root.

SECTION IX.

Of Characters *and their* Explanation.

In the following Sheets of this Treatiſe, there are ſeveral Characters and Expreſſions uſed in order to ſhorten the Work which I ſhall here explain.

I. Wherever you meet with this Sign +, (*more*) it ſignifies that the Number following the Sign is to be added to the Number going before it; thus 4 + 8, is read 4 more 8, and ſignifies, that 8 is to be added to 4.

II. This

II. This Sign — (*less*) signifies that the Number following it, is to be subtracted from the Number going before it; thus 6 — 2 is read 6 less 2, and signifies, that 2 is to be taken from 6.

III. This Sign × (*into*) signifies Multiplication, and implies that the Numbers this Sign is between, are to be multiplied together; thus 4 × 9 imports, that 4 is to be multiplied by 9; and 2 × 3 × 6 × 5, signifies that 2 is to be multiplied by 3, and that Product by 6, and this Product by 5, and the like of any other.

IV. This Sign ÷ (*by*) signifies Division, and shews that the Number going before the Sign, is to be divided by the Number following it; thus 12 ÷ 4 implies, that 12 is to be divided by 4; but Division is most commonly expressed by setting down the Dividend or Number to be divided, and placing the Divisor or dividing Number under it, with a Line drawn between them, like a vulgar Fraction $\frac{12}{4}$ implies that 12 is to be divided by 4, and if 48,327 was to be divided by 2,15, I express it thus $\frac{48,327}{2,15}$ &c.

V. This Sign = (*equal*) signifies that the Numbers or Expressions on each side thereof are equal one to the other; thus 4 + 8 = 12, signifies that 8 added to 4 is equal to 12; and 6 — 2 = 4 implies that 2 taken from 6 leaves 4, or 6 lessened by 2 is equal to 4; and 4 × 9 = 36 implies that 4 multiplied by 9, gives a Product equal to 36; and $\frac{12}{4} = 3$, signifies that 12 divided by 4, gives a Quotient equal to 3, and the like of other Expressions.

VI. The Terms of Proportions are expressed by certain Points between the Terms; thus 4 : 6

: : 10 : 15, and is read, as 4 is to 6, ſo is 10 to 15, ſo that the two Points : between the two firſt Terms is read, *is to*; the four Points : : between the ſecond and third Terms is read, *ſo is*, and the two Points : between the third and fourth Terms is read, *to*.

TAKE an Example where all the forementioned Characters are uſed : Suppoſe I buy 120 Eggs at two a Penny, and 120 more at three a Penny, and ſell them again at five for two Pence; whether do I loſe or gain, and how much?

$\frac{120}{2} = 60$ *d.* the Price the firſt 120 coſt, and $\frac{120}{3} = 40$ *d.* the Price that the ſecond 120 coſt; and $60 + 40 = 100$, the Price the 240 coſt. Then as 5 *egg.* : 2 *d.* : : 240 *egg.* : 96 *d.*; for $240 \times 2 = 480$, and $\frac{480}{5} = 96$ *d*, the Price the Eggs were ſold at; and 100 *d.* — 96 *d.* = 4 *d.* the Money loſt.

WHEN ſeveral Terms are connected together by a Line drawn over them, it implies that the Reſult of thoſe Terms, being ordered, or operated, as the Signs between them denote, is to be taken for the ſucceeding part of the Work; for Example, ſuppoſe an Expreſſion appeared thus; $\overline{6 \times 2} - 4$, it implies that 6 is to be multiplied by 2, and from the Product 12, 4 is to be taken:

Again ſuppoſe this Expreſſion $\overline{\overline{\frac{\overline{8 \times 3 + 9}}{11}} - 1{,}5} \times 8$ occurred; it imports, that 8 is to be multiplied by 3, and to the Product 24 is to be added 9, and the Sum 33 is to be divided by 11, and from the Quotient 3, 1,5 is to be taken, and the Remainder 1,5 is to be multiplied by 8.

AND in the ſame Manner other Expreſſions of a like kind are to be underſtood.

MENSURATION

MENSURATION.

PART I.

SECTION I.

MENSURATION is the Method whereby the Contents of Superficies and Solids are found, by having their Dimensions given in any ſort of Meaſure; and conſiſts of three Parts; *viz. Lineal*, *Superficial*, and *Solid*.

I. *Lineal Meaſure* is nothing more than the meaſuring of Lengths, and is uſed in meaſuring the Dimenſions of ſuperficial and ſolid Figures.

II. *Superficial Meaſure* is the finding the Number of ſquare Inches, Feet, Yards, *&c.* in any flat Figure, as a Floor, a Wall, *&c.* by having the Dimenſions in Lineal Meaſure.

III. *Solid Meaſure*, is the finding the Number of Cubic, or ſolid Inches, Feet, Yards, *&c.* in thoſe Figures that have Length, Breadth, and Depth, as a Block of Stone, Marble, *&c.* by having the Dimenſions in Lineal Meaſure.

As the chief Uſes of Lineal Meaſure are, to take the Dimenſions of ſuperficial and ſolid Figures; there need no more be ſaid of it, than,

12 Inches - are -	1	Foot.
3 Feet - - - - - -	1	Yard.
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ Feet -	1	Pole, Perch or Rod.
40 Poles - - - - -	1	Furlong.
8 Furlongs - - - -	1	Mile.

Superficial and *Solid Measure* I ſhall treat of ſeparately; and firſt of

SUPERFICIAL MEASURE.

WHEREIN it will be neceſſary to lay down,
First, *The uſeful Definitions, and Problems.*
Secondly, *The common Methods of calculating uſed by Artificers.* And
Thirdly, *The Methods of computing the Areas of various ſorts of Figures.*

SECTION II.

DEFINITIONS.

I. A LINE is Length without Breadth; and is either right [ſtrait] when it is the ſhorteſt Diſtance between two Points; or curved [crooked] when it is not the ſhorteſt Diſtance between two Points.

II. A Superficies is a Figure which hath Length and Breadth, and is incloſed or contained between right or curved Lines.

Note,

Note, One curved Line may contain a Space or Superficies; but of right Lines, leſs than three cannot contain a Space.

III. When one Line is inclined towards another Line in ſuch a Manner, as if either or both were continued, they would meet; then the Opening of theſe Lines is called, *an Angle*. *Fig.* 1. *Pl.* 1.

IV. When one Line ſtandeth ſo on another, as to incline to neither ſide; but maketh the Angles on each ſide equal; each of thoſe Angles is called *a right one*; and the Line ſo ſtanding on the other, is called, *a Perpendicular*, to that whereon it ſtandeth. *Fig.* 2.

V. All three ſided Figures are called, *Triangles*, but admit of the following Diſtinctions:

Firſt, If the three ſides are unequal, it is called *a Scalene Triangle*. *Fig.* 3.

Secondly, If the three ſides are equal, is is called *an Equilateral Triangle*. *Fig.* 4.

Thirdly, If only two ſides are equal, it is called *and Iſoſceles Triangle*. *Fig.* 5.

Fourthly, If it hath one right Angle, it is called *a Right-angled Triangle*. *Fig.* 6.

VI. All four ſided Figures are called *Quadrilaterals*, but admit of the following Diſtinctions:

Firſt, When the four ſides are equal, if the Angles are right ones, it is called *a Square*; *fig.* 7. but if the Angles are not right ones, it is called *a Rhombus*. *Fig.* 8.

Secondly, When only two ſides are equal, if the Angles are right ones, it is called *a Rectangle*; *fig.* 9. but if the Angles are not right ones, it is called *a Rhomboides*. *Fig.* 10.

Note. These four are called *Parallelograms*, as having their oppoſite Sides paralled or equidiſtant

to

to each other; but all other four sided Figures are called *Trapezia*. *Fig.* 11.

VII. A CIRCLE is a plain Figure, contained under, or bounded by one Line called *the Circumference*; to which all right Lines drawn from a certain Point within the Figure, called *it's Center*, are equal. *Fig.* 12.

VIII. THE Diameter of a Circle is a right Line drawn through the Center, and terminated at each End by the Circumference, and divides the Circle into two equal Parts, each called *a Semicircle*.——*Note*, Half the Diameter is called *the Radius*. *Fig.* 12.

IX. THE Chord of a Circle is a right Line drawn within the Figure, not through the Center, and terminated at each End by the Circumference, and divides the Circle into two unequal parts, called *Segments*. *Fig.* 12.

X. IF the Chord of a Circle cut the Diameter at right angles, that Part of the Diameter lying between the Chord and Circumference is called *a versed Sine*, and is the height of the Segment. *Fig.* 12.

XI. A SECTOR is a Figure contained under two Radii of a Circle, and that part of the Circumference included between the two Radii. *Fig.* 12.

XII. A POLYGON is a Figure contained under many Sides; if a Circle will pass through all it's Angles, it is called *a regular Polygon*, otherwise *an irregular one*.——*Note*, a Polygon is named according to the Number of Sides it hath; *viz.* If it has 5 Sides, it is called *a Pentagon*; if 6 Sides, *a Hexagon*; if 7, an *Heptagon*; if 8, *an Octagon*; if 9, *a Nonagon*; if 10, *a Decagon*; if 11, *an Undecagon*; if 12, *a Duodecagon*. See *Fig.* 13, 14, 15, 16, 17, 18, 19, 20.

XIII. IN

XIII. In any Quadrilateral, if a Line be drawn to any two oppoſite Angles, that Line is called *a Diagonal*. *Fig.* 21.

XIV. The Altitude or Height of any Figure, is a Perpendicular, let fall from the higheſt Point of the Figure to it's Baſe, or Side oppoſite, which is the Line you ſuppoſe the Figure to ſtand on.

XV. The Area of any Figure, is the ſuperficial Content.

SECTION III.

As there is ſometimes a Neceſſity of letting fall a Perpendicular, in order to come at the Area of a Figure; it is therefore convenient to know how to ſolve the following Problems.

PROBLEM I.

To biſect, or divide into two equal parts the Line A B.

CONSTRUCTION.

Set one Foot of the Compaſſes on the end B, and open the other to any convenient Diſtance greater than half AB, and with that Opening deſcribe the Arch DE; then ſet one Foot on A, and with the ſame Opening croſs the former Arch in D and E; and draw the Line DE, which will biſect the given Line AB in the Point C. *Fig.* 22.

PRO-

PROBLEM II.

On any Point C of a given Line AB, to erect a Perpendicular

CONSTRUCTION.

On any convenient Point as D, out of the given Line, set one Foot of the Compasses, and extend the other to the Point C, with that Extent describe a Circle; and from the Point E, where it cuts the given Line AB, draw the Diameter EDF, from the Point C, and through the Extremity of the Diameter F, draw the Line CF, which will be perpendicular to the given Line AB, and stand on the Point C, as was required. *Fig*. 23.

PROBLEM III.

To let fall a Perpendicular to any given Line AB, from a Point C given above the Line.

CONSTRUCTION.

Set one Foot of the Compasses on the Point C, and with any convenient Opening describe the Arch DE, on the Points D and E; with the same Opening describe Arches below the Line, to cross each other in F, lay a Ruler by C and E, and draw the Line CD, which will be perpendicular to the Line AB, as was required. *Fig*. 24.

SEC-

SECTION IV.

THE Menſuration of all right lin'd Figures may be obtained by the help of the two following Propoſitions.

PROPOSITION I.

HAVING the Length and Breadth of a right lined Parallelogram given, to find the Area thereof.

RULE.

MULTIPLY the Length by the Breadth, [or perpendicular Height] and the Rectangle [or Product] is the Anſwer.

PROPOSITION II.

HAVING the Baſe and perpendicular Height of any right lined Triangle given, to find the Area.

RULE.

MULTIPLY the Baſe by half the perpendicular Height, or the perpendicular Height by half the Baſe; and the Rectangle is the Area.

ARTIFICERS, by the help of theſe two Propoſitions compute the Area of all right lined Figures, and that by Numbers after three Methods, *viz.*

 Firſt,

First. By Aliquot Parts.
Secondly. By Decimals. And
Thirdly. By Duodecimals.

But the latter of theſe are moſtly uſed when the Dimenſions are taken in Feet, Inches, *&c.*

Different Artificers compute the Area of their Work by different Meaſures, *viz.*

First. By the Foot; as Glazing.

Secondly. By the Yard; as Painting, Plaiſtering, Paving, *&c.*

Thirdly. By the Square of 100 Feet; as Flooring, Partitioning, Roofing, Tyling, *&c.*

Fourthly. By the Rod of $16\frac{1}{2}$ Feet, whoſe Square is $272\frac{1}{4}$, by which Bricklayers compute their Work.

There is a Table alſo uſeful for Land-meaſure, which is as follows:

Land is beſt meaſured by the *Gunter*'s Chain, which is 4 Poles long, and is divided into 100 Links, each being 7,92 Inches, and as 40 Poles in Length, and 4 in Breadth make a Statute Acre; therefore

625 ſquare Links = 1 Pole
100000 - - = 160 = 1 Acre = 4840 Yards.

SECTION V.

QUESTION I.

What is the Area of a Window, whoſe Length is 14 *Feet* 6 *Inches*, and Breadth 4 *Feet* 9 *Inches*?

This Example I ſhall exemplify by the three Methods of Operation.

First,

First. By Aliqout Parts. *In.*			*Secondly.* By Decimals.	*Thirdly.* By Duodecimals.
6	$\frac{1}{2}$	14 - 6	14,5	14 - 6
		4	4,75	4 - 9
		58 - 0	725	58 - 0
3	$\frac{1}{2}$	7 - 3	1015	10 - 10 - 6
		3 - 7$\frac{1}{2}$	580	68 - 10 - 6
		68 - 10$\frac{1}{2}$	68,875	

IN all theſe different Methods of Operation, the Reſult turns out the ſame exactly; which is 68 ſquare Feet, 10$\frac{1}{2}$ ſquare Inches.

BUT the method by Duodecimals being moſtly in Uſe among Workmen, I ſhall ſhew how it is performed: The other Methods being obvious from common Arithmetic.

DUODECIMALS has it's Name from *Duodecem* [Latin for 12] and by it the ſeveral Denominations of Meaſure are ſuppoſed to be divided each into twelve Parts.

Therefore 1 Foot = 12 Inches.
1 Inch = 12 Parts,
1 Part = 12 Seconds, *&c.*

BUT as all Superficies are compoſed or made up of the Squares of the ſmalleſt Denomination in one Dimenſion, as often repeated as are the Number of Parts of the like Denomination in the other Dimenſion.

Therefore, 1 ſquare Foot = 144 ſquare Inches,
1 ſquare Inch = 144 ſquare Parts,
1 ſquare Part = 144 ſquare Seconds, *&c.*

AND hence comes the Rule for Duodecimals, *viz.*

Feet multiplied	by	Feet		Produces	Feet,
Feet -	D°. -	Inches			Inches,
Feet -	D°. -	Parts			Parts, &c.
Inches -	D°. -	Inches			Parts,
Inches -	D°. -	Parts			Seconds, &c.
Parts -	D°. -	Parts			Thirds, &c.

ONLY obſerving, that in the Multiplication of any of the Denominations excepting Feet by Feet, if the Product be above 12, carry as many Units as there may be found 12's in that Product, to the Product of the next ſuperior Denomination; and ſetting down in every Denomination (excepting Feet) ſo much of the Product as is under 12, or exceeds 12 or 12's.

As in the foregoing Example, *viz.* 14 Feet 6 Inches to be multiplied by 4 Feet 9 Inches.

F.		*I.*		
14	-	6		
4	-	9		
58	-	0		
10	-	10	-	6
68	-	10	-	6

FIRST I ſay, 4 times 6 is 24, which are two 12's, and nothing over, therefore I ſet down 0 under the Inches, and carry 2, ſaying 4 times 4 is 16, and 2 I carry'd is 18, which is 8 and carry 1; 4 times 1 is 4 and 1 I carry'd makes 5, which makes 58 Feet: Again, 9 times 6 is 54, which are four 12's, and 6 over, which 6 I ſet down under the Place of Parts, and carry 4, ſaying 9 times 4 makes 36, and 4 I carry'd makes 40, which is 0 and carry 4, and 9 times 1 makes 9, and 4 I carry'd makes 13, which makes this Product 130, in which are ten 12's, and 10 over, therefore I

ſet

ſet down 10 under the Place of Inches, and as the Multiplication is ended, and nothing to carry the ten Twelves to, I ſet them down under the Place of Feet, and then add the Products together.

BUT here you are to note, that in the foregoing Operation, when I ſaid 4 times 6 is 24; 24 what? Why, 24 times 12 ſquare Inches, which is twice 144 ſquare Inches, for ſo many makes 2 ſquare Feet, which is what was carry'd to the Product of Feet. And when I ſaid 9 times 6 is 54, that is 54 times 12 ſquare Parts, that is 4 times 12 ſquare Inches, and 6 times 12 ſquare Parts; and the Product 130 is 130 times 12 ſquare Inches, which is 10 times 144, and 10 over, that is 10 ſquare Feet, and 10 times 12 ſquare Inches. And the like is to be underſtood in other Operations. But it will be more obvious perhaps to ſet the loweſt Denomination of the multiplying Dimenſion one Denomination more towards the right hand. Thus, I put the 9 Inches under the Place of Parts, and multiply by the 4 Feet as before directed.

F.	I.	P.
14 -	6 -	0
	4 -	9
58 -	0 -	0
10 -	10 -	6
68 -	10 -	6

Now it will often happen that the Feet in the given Dimenſions are ſo many, that in order to multiply them by the leſſer Denominations, and take a twelfth of their Product as before directed, there is a Neceſſity of doing ſome Work upon a ſpare Piece of Paper; to prevent which do thus.

Divide the Feet you are going to multiply with by 12, reserving the Quote and Remainder in your Mind. Then multiply the Remainder by the lesser Denominations, and set down the Overpluss of 12's under the Inches, and multiply the Quote by the same Dimension, adding to the Quote what you carry'd, and it gives the Feet.

As in the foregoing Example I say, 12 in 14 goes 1, and 2 remains; then in multiplying by the 9 Inches, I say, 9 times 6 is 54, that is 6 and go 4, setting the 6 under the Place of Parts; then 9 times 2 (the Remainder) is 18, and 4 carry'd is 22, that is 10, and go 1, set the 10 under the Inches, and carry 1; then 9 times 1 (the Quote) is 9, and 1 carry'd makes 10, which set under the Feet; and adding the several Products together, as before directed, ends the Work.

The Use of dividing the Feet thus by 12, and working with the Quote and Remainder, will appear more conspicuous in those Operations where the Feet are many, as it will be evident by comparing one Example wrought both ways.

EXAMPLES.

What will the Product be, if 368 Feet 10 Inches be multiply'd by 137 Feet 8 Inches?

In order to work this Example in the most ready manner, I operate thus:

F.	*I.*	*P.*
368 -	10 -	0
	137 -	8
245 -	10 -	8
114 -	2	
2576		
4784		
50776 -	00 -	8

The

THE 12's in 368 are 30, and 8 remains, which Quote and Remainder I reserve in my Mind, and say, 8 times 10 is 80, that is 8 and carry 6, the 8 I place under the Parts, and say 8 times 8 (the Remainder) is 64, and 6 carry'd make 70, which is 10 and carry 5, the 10 I place under the Inches, and say, 8 times 30 (the Quote) is 240, and 5 carry'd, make 245, which I set under the Feet; and so the multiplying with the 8 Inches is done. Again, the 12's in 137 are 11, and 5 remains, which Quote and Remainder I reserve in my Mind, which multiply by the 10 Inches; saying, 10 times 5 (the Remainder) is 50, which is 2 and carry 4, the 2 I put under the Inches, and say, 10 times 11 is 110, and 4 carry'd is 114, which I set under the Feet; and so the multiplying 137 Feet by the 10 Inches is done. Lastly, I multiply the 368 Feet by the 137 Feet, setting down the Product as you see in the Example, and adding the several Products together, the Work is finished.

THERE is very seldom any Occasion for multiplying Feet, Inches, and Parts; but if at any time you meet with a Necessity for so doing, you will very easily know how, by carefully examining this Example as here wrought:

F.	I.	P.	S.	T.
39 -	10 -	7 -	0 -	0
	18 -	8 -	4 -	0
1 -	1 -	3 -	6 -	4
26 -	7 -	0 -	8	
0 -	10 -	6		
15 -	0			
702				
745 -	6 -	10 -	2 -	4

SECTION VI.

Of Artificers Works.

FIRST, by the Foot; as Glazing, and Masons flat Work.

QUESTION II.

WHAT will the glazing a Triangular Skylight come to at 10 *d. per Foot*, supposing the Base 12 Feet 6 Inches long, and the perpendicular Height 16 Feet 9 Inches?

By *Duodecimals*.

F. I.

16 - 9 - = the Height.

6 - 3 = $\frac{1}{2}$ the Base.

4 - 2 - 3

100 - 6

104 - 8 - 3 = the Area.

52

34 - 8

0 - $6\frac{3}{4}$

20) 8,7 - $2\frac{3}{4}$

4 - 7 - $2\frac{3}{4}$ ANSWER, 4 *l.* 7 *s.* $2\frac{3}{4}$ *d.*

d. & *d.* | $\frac{1}{2}$, $\frac{1}{3}$

6 4

1 *d.*

6 | $\frac{1}{2}$ | 10

2 | $\frac{1}{3}$ | 5

$1\frac{3}{4}$

$6\frac{3}{4}$

By

By *Decimals.*

$$
\begin{array}{rl}
16{,}75 & \\
\underline{6{,}25} & \\
8375 & \\
3350 & \quad d. \qquad \pounds. \\
10050 & \quad 10 = {,}401\cancel{0} \\
\hline
104{,}6875 & \\
\underline{\cancel{6}\ 140{,}0,} & \text{the Multiplier} \\
9)\ \underline{628} & \quad \text{inverted.} \\
69\cancel{1} & \\
1047 & \\
\underline{41875} & \\
\underline{4{,}3619\cancel{1}} &
\end{array}
$$

If what has been already ſaid in the Decimal Part be well underſtood, it will be no difficult Matter to ſee (immediately) how this Decimal Operation is performed, and alſo thoſe which follow; the Area given by the Duodecimal Work, is valued by Aliquot Parts or Practice.

QUESTION III.

There is a Houſe with three Tier of Windows, three in a Tier, the Heights are, *viz.* Firſt Tier 7 Feet, 10 Inches.——Second, 6 Feet, 8 Inches.——And third 5 Feet, 4 Inches. And the Breadth of each 3 Feet, 11 Inches; what will the Glazing come to at 14 *d. per Foot?*

By

By *Duodecimals.*

	F.	*I.*	
Add	7 -	10	
	6 -	8	
	5 -	4	
	19 -	10	
Multiply by	—	3	
	59 -	6	
		3 -	11
	54 -	6 -	6
	178 -	6	
	233 -	0 -	6 Parts.

d.

2 | $\frac{1}{6}$ | 233 — 6 is $\frac{1}{24}$ | 14 *d.* — $\frac{1}{2}$

38 - 10

$\frac{1}{2}$

20) 27,1 - 10$\frac{1}{2}$ — ANSWER,

13 - 11 - 10$\frac{1}{2}$ — *l.* *s.* *d.* 13-11-10$\frac{1}{2}$

By *Decimals.*

59,5

3,91$\cancel{6}$

9) 3570 — *d.* £.

396$\cancel{6}$ — 14 = ,058$\cancel{3}$

595

5355

1785

233,041$\cancel{6}$

$\cancel{3}$850,0

9) 699

776

18643

116521

13,5940

To

To solve this Question, I first add together the Heights of the Windows over each other, and then multiply the Sum by 3 the Number of Rows, which gives a Length equal to the nine Windows set on each other, and this multiply'd by the Breadth gives the Area.

Glaziers generally measure their Work to a Quarter of an Inch; and never make any allowances for round or oval Windows, but always measure them to the greatest Length; for there is more Trouble in cutting the Glass to those Shapes, than the Value of the Glass omitted.

QUESTION IV.

What is a Marble Slab worth, whose Length is 5 Feet 7 Inches, and Breadth 1 Foot 10 Inches, at 6 *s. per Foot?*

	By *Duodecimals.*	By *Decimals.*
	5 - 7 - 0	5,58$\dot{3}$
	1 - 10	1,8$\dot{3}$
	4 - 7 - 10	9) 16750
	5 - 7	18611
s.	10 - 2 - 10	44666
5 | $\frac{1}{4}$	2 - 10	55833
1 | $\frac{1}{3}$	10	10,2361 s.
	1 - 5	,3 = 6
	3 - 01 - 5	3,07083
	Answer,	
	3 *l.* 1 *s.* 5 *d.*	

l.		s.
2	$\frac{1}{6}$	6
p.		
6	$\frac{1}{4}$	1-0
3	$\frac{1}{2}$	3
1	$\frac{1}{3}$	1 $\frac{1}{2}$
		$\frac{1}{2}$
		1-5

Secondly.

Secondly. By the Yard; as Paviors, Painters, Plaisterers, and Joyners.

RULE.

DIVIDE the Feet (found as before) by 9 [because 9 square Feet are 1 Yard] and it gives the Area.

QUESTION V.

WHAT will the paving a Yard of a rectangular Form come to at 3 *s.* 2 *d per Yard*; supposing the Length 27 Feet, 10 Inches, and the Breadth 14 Feet, 9 Inches?

By *Duodecimals*.

27 - 10
14 - 9
20 - 10 - 6
11 - 8
387
9) 410 - 6 - 6
45 - 5 - 6 6 at 3 - 2 *s.* *d.*

s. 2 | $\frac{1}{10}$ | 45
s. 1 | | 4 - 10
d. 2 | $\frac{1}{2}$ | 2 - 5
d. | $\frac{1}{6}$ |
7 - 6
1 - $10\frac{3}{4}$
7 - 4 - $4\frac{3}{4}$

F. | | *s.* *d.*
3 | $\frac{1}{3}$ | 3 - 2
1 | $\frac{1}{3}$ | 1 - $0\frac{1}{2}$
I.
6 | $\frac{1}{2}$ | 4, $4\frac{1}{4}$ } 2 *F.*
P.
6 | $\frac{1}{12}$ | 2
1 - $10\frac{1}{2}$

ANSWER, 7 *l.* 4 *s.* $4\frac{3}{4}$ *d.*

By

By *Decimals*.

```
        27,83
        14,74
       ------
        13916
       194833
      1113333
      2783333          s.  d.
     --------          3 - 2 = ,1583
  9) 410,5416
     --------
      45,615740
      38 51,0
   9) 137
      ------
        152
       3649
      22808
      45616
     -------
     7,2225
```

QUESTION VI.

I HAVE paved a rectangular Court-Yard 42 Feet, 9 Inches in Front; and 68 Feet, 6 Inches in Depth: And in this I have laid a Foot-Way the Depth of the Court, of 5 Feet, 6 Inches in Breadth: The Foot-Way is laid with Purbeck Stone at 3 *s.* 6 *d. per Yard*, and the Rest with Pebbles at 3 *s. per Yard*; what will the whole come to?

Find the Value of the whole Court at 3 *s.* and the Foot-Way at 6 *d.* and these Values added together will give the whole Cost.

68 - 6
42 - 9

51 - 4 - 6
21 - 0
136
272

9) 2928 - 4 - 6

325 - 3 - 4 6

The Court's Area.

68 - 6
5 - 6

34 - 3 - 0
342 - 6

9) 376 - 9

41 - 7 - 9

The Foot-Way's Area.

s.		*Yds.* *F.* *I.* *P.* *s.*
2	$\frac{1}{10}$	325 - 3 - 4 - 6 at 3
1	$\frac{1}{2}$	32 - 10
		16 - 5
		1 - $1\frac{1}{4}$
		l. 48 - 16 - $1\frac{1}{4}$
d.		*Yds.* *F.* *I.* *d.*
6	$\frac{1}{40}$	41 - 7 - 9 at 6
		1 - 0 - 6
		5
		l. 1 - 0 - 11

F. *I.* *P.*		*s.* *d.*
3 - -	$\frac{1}{3}$	3 - 0
4 -	$\frac{1}{9}$	1
6	$\frac{1}{8}$	- $1\frac{1}{4}$
		1 - $1\frac{1}{4}$
F. *I.*		*s:* *d.*
3 - -	$\frac{1}{3}$	0 - 6
		2 } 6
1 - -	$\frac{1}{3}$	2 } Feet.
9		$0\frac{1}{2}$
		$0\frac{1}{2}$
		0 - 5

	l.	*s.*	*d.*
The whole Court at 3 *s.* is	48 —	16 —	$01\frac{1}{4}$
The Foot-Way at 6 *d.* is -	1 —	0 —	11
The whole Cost ——— *l.*	49 —	17 —	$00\frac{1}{4}$

QUESTION

QUESTION VII.

WHAT will the plaistering a Ceiling at 10 *d. per Yard* come to; supposing the Length 21 Feet, 8 Inches, and the Breadth 14 Feet, 10 Inches?

By *Duodecimals*.

21 - 8
14 - 10
18 - 0 - 8
9 - 4
294

d. *d.* 9) 321 - 4 - 8 *d.*
6 *and* 4 | $\frac{1}{2}, \frac{1}{3}$ | 35 - 6 - 4 - 8 at 10
17 - 6
11 - 8
$6\frac{3}{4}$

20) 29 - $8\frac{3}{4}$
1 - 9 - $8\frac{3}{4}$

ANSW. 1 *l.* 9 *s.* $8\frac{3}{4}$ *d.*

F. | | *d.*
3 - | $\frac{1}{3}$ | 10
I. | | $3\frac{1}{4}$ } 6 *F.*
4 - | $\frac{1}{9}$ | $3\frac{1}{3}$ }
$\frac{1}{4}$
$6\frac{3}{4}$

By *Decimals.*

14,83
21,6
9) 8900
9188
1483 *d.* £.
29666 10 = ,0416
9) 321,38
35,7098, &c.
6140,0
9) 214
238
357
14283
1,4878

PLAISTERERS Works are principally of two Kinds, namely, *first*, Works lath'd and plaister'd, which is call'd *Ceiling*. *Secondly*, Works rendered, which is of two Kinds, *viz.* upon Brick Walls, or between Quarters in the Partition between Rooms.

IN meaſuring Rendering upon Brick Walls, there are no Deductions made ; but in meaſuring Rendering between Quarters, there is commonly a fifth Part of the whole Area deducted : But when Rendering between Quarters is whited or colour'd, there is commonly a fourth or fifth Part added to the whole Area, for the Sides of the Quarters and Braces, &c.

Note, YOU muſt make Deductions for Doors, Windows, &c.

QUESTION

QUESTION VIII.

THERE is a Quantity of Partitioning that measures 234 Feet, 8 Inches about, and 14 Feet, 6 Inches high; but is Rendered between Quarters: The Lathing and Plaistering will be 8 *d. per Yard*, and the Whiting 2 *d. per Yard*; what will the whole come to?

By *Duodecimals*.		By *Decimals*.
234 - 8		234,6
14 - 6		14,5
177 - 4 - 0		11733
9 - 4		93866
3276		234666
9) 3402 - 8		9) 3402,6
5) 378 - 0 - 8	The Area in Yards.	5) 378,074
75 - 5 - 6	This $\frac{1}{5}$ Part deduct.	75,6148
302 - 4 - 2	Rem. the Ar. plaister'd.	302,4592
378 - 0 - 8	To the Area in Yards	378,074
75 - 5 - 6	Add the $\frac{1}{5}$ Part.	75,6148
453 - 6 - 2	Gives the Ar. whited.	453,6888

302,4592	453,6888	
8 *d.* = ,03 *l.*	2 *d.* = ,0083 *l.*	
10,081973	1512296	10,081973
l. *s.* *d.*	36295104	3,64463336
10—1—7½	3,64463336	13,72660636
	l. *s.* *d.*	
	3—12—10¾	

ANSWER, 13 *l.* 14 *s.* 6¼ *d.* the whole Coſt.

IN theſe two Operations, inſtead of multiplying by 3, and dividing by 9, as directed; I take $\frac{1}{3}$ of the Multiplicand, which is exactly the ſame, tho' more expeditious.

QUESTION IX.

SUPPOSE a Room that was painted at 8 *d.* *per Yard*, meaſures as follows: The Height (taking in the Cornice and Mouldings) is 11 Feet, 7 Inches; the Girt or Compaſs 74 Feet, 10 Inches; the Door 7 Feet, 6 Inches, by 3 Feet, 9 Inches; five Window-Shutters, each 6 Feet, 8 Inches, by 3 Feet, 4 Inches; the Breaks in the Windows 14 Inches deep, and 8 high; the Chimney 6 Feet, 9 Inches by 5 Feet; a Cloſet the Height of the Room 3½ Feet deep, and 4¾ Feet in Front, with Shelving together 22 Feet, 6 Inches by 10 Inches; the Shutter, Door, and Shelves, are coloured on both Sides: What will the whole come to?

74,83

7 4,8 3 × 1 1,5 8 3 =	—	866,7308	The Area of the whole Room.
6,6 × 3,3 × 5 =	— —	111,1111	The Area of the Shutters once.
$\overline{8+8+3,3+3,3}$ = 22,6 × 1,16 × 5 =		132,2222	The Breaks in the Windows.
7,5 × 3,7 5 =	— —	28,125	The Door once.
$\overline{\overline{3,5 \times 2} + \overline{4,75 \times 2}}$ = 16,5 × 11,583 =		191,125	The Closet's Area.
2 2,5 × ,8 3 × 2 =	— —	37,5	The Area of the Shelves.
The Sum of the Areas	— — —	1366,8138	Out of which deduct.
6,7 5 × 5 =	— —	33,75	The Chimney's Area.
Remains	— — — — —	1333,0638	The Area of the whole Work.

Then $\frac{1333,0638}{9}$ = 148,1182, *Yds.* &c. then if 1 *yd.* : ,03 *l.* : : 148,1182 *yd.* : 4,93727 *l.* &c. = 4 *l.* 18 *s.* 9 *d.*

PAINTERS

PAINTERS take their Dimensions with a String, and measure from the Top of the Cornice to the Floor, girting the String over all the Mouldings and swelling Pannels: And in measuring of Doors, they account the Height and Breadth of the Door so much more, as in the thickness of the Stuff; it being reasonable they should be paid for all Places whereon their Colour is laid. Their Price they generally proportion according to the Number of Times they lay their Colour on.

Note, There must always Deductions be made for Chimnies, Casements, &c. if any within the Dimensions they take.

QUESTION X.

WHAT will the Wainscotting a Room come to at 6 *s. per square Yard*; supposing the Height of the Room (taking in the Cornice and Mouldings) is 12 Feet 6 Inches, and the Compass is 83 Feet 8 Inches; the Window Shutters each 7 Feet 8 Inches, by 3 Feet 6 Inches, and the Door 7 Feet by 3 Feet 6 Inches; the Shutters and Door being worked on both Sides is reckoned Work, and half Work?

```
   F.  I.  P.
  83 -  8 - 0
       12 - 6
 ---------------
  41 - 10 - 0
 104 - 00
 ---------------
Add { 145 - 10  Room's Area.
    {  52 -  6 = ½ Do. & Sh.Do.
 ---------------
Sum   198 -  4 = whole W.
 ---------------
```

```
  F.  I.  P.
  7 -  8
       3 - 6
 ---------------
  3 - 10 - 0
 23 -  0
 ---------------
 26 - 10
       3  Windows
 ---------------
 80 -  6 = Shut.once
 ---------------
```

```
   F.    I.    yds.   F.  I.
9) 198 - 4 (22 - 0 .. 4

Then { 22
     { ,3 = 6 s.
     { ---------
     {  6 - 12
        - - - - 2½
     ---------------
        6 - 12 - 2½
```

```
  F.  I.
  3 - 6
      7
 ---------------
Add { 24 -  6 the Door once.
    { 80 -  6 the Shutters once.
 ---------------
2) 105 - 0
 ---------------
    52 - 6 the ½ Door and Shut.

F.          s.  d.
4 = 1/27 | 6 - 0
         | ---------
         |     2½
         ---------------
            l.  s.  d.
ANSWER,     6  12  2½
```

JOYNERS meaſure their Work in Height with a String, and their Length or Compaſs upon the Floor as Painters do; for they ſay, they ought to meaſure where their Plane touches, therefore they take the Cornice and Mouldings into the Height of the Room.

Note. THEY only take the Cornice and Mouldings in with the Height of the Room, when they are ſtruck with a Horſe-plane, (as they call it) but if they are wrought by Hand, then they are paid ſo much *per Foot* running Meaſure.

ALL Stuff an Inch and half thick and under, wrought on both Sides, is by them reckoned at Work and Half Work; but Stuff of greater Thickneſs wrought on both Sides, is valued at Double Work.

THEY make Deduction for all Vacancies that fall within their Work; and Window Boards, Sofite Boards, Cheeks, *&c.* are meaſured by themſelves.

Thirdly, BY the Square: As, Flooring, Partitioning, Roofing, Tyling, *&c.* ſuch Works are meaſured by a ten Feet Rod, whoſe Square is 100. Therefore

RULE.

DIVIDE the Feet, (found as before) by 100, and it gives the Area.

QUESTION XI.

SUPPOSE a Houſe of three Stories, beſides the Ground-floor, was to be floor'd at 6 *l.* 10 *s. per Square*; the Houſe meaſures 20 Feet 8 Inches, by 16 Feet 9 Inches; there are 7 Fire-places, whoſe Meaſures are as follows: Two each 6 Feet, by 4 Feet 6 Inches; two other, each 6 Feet, by 5 Feet 4 Inches; and two each 5 Feet 8 Inches, by 4 Feet 8 Inches; and the ſeventh 5 Feet 2 Inches, by 4 Feet; and the Well-Hole for the Stairs is 10 Feet 6 Inches, by 8 Feet 9 Inches; what will the Whole come to? 4 - 6

4 — 6   5 — 4   5 — 8   5 — 2   10 - 6

6   6   4 — 8   4   8 - 9

27 — 0   32 — 0   3 — 9 — 4   20 — 8   7 - 10 - 6

2   2   22 — 8   84

54   64   26 — 5 — 4   91 - 10 - 6

2   4

20 - 8   52 — 10 — 8

16 - 9

15 - 6 - 0

330 - 8

346 - 2

4

1384 - 8 The Ar. of the four Floors.

559 - 0 - 8 The Ar. of the Deduct.

100) 825 - 7 - 4 The Area of the Work.

S. F. I. P. l. s.

8 — 25 — 7 — 4 at 6 — 10 *per Square.*

l. s. d.

Answer, 53 — 13 — 3¼

367 - 6 - 0 The Well Hole.

54 - 0 - 0 The first Chimnies.

64 - 0 - 0 The second Chimnies.

52 - 10 - 8 The third Chimnies.

20 - 8 - 0 The fourth Chimney.

559 - 0 - 8 The whole Deductions.

s.	Sqrs.			l. s. d.
10 - ½	8	25 *F.*	¼	6 - 10 - 0
	6			
	48	6 *I.*	$\frac{1}{50}$	1 - 12 - 6
	4	1	⅛	7¼
	1 - 13 - 3¼	4*P.*	⅓	1¼
				¼
	l. 53 - 13 - 3¼			1 - 13 - 3¼

In

In Flooring there are Deductions made alſo for Hearth-Stones, excepting the Hearth Stone hath a Border round it; and then the Hearth is meaſured in with the Floor.

QUESTION XII.

In 173 Feet 10 Inches in Length, and 10 Feet 7 Inches in Height of Partitioning; how many Squares?

By *Duodecimals.*

```
      173 - 10
            10 - 7
      ------------
      101 -  4 - 10
     1738 -  4
100) 1839 -  8 - 10
S.      F.    I.   P.
18 — 39 — 8 — 10
```

By *Decimals.*

```
      1 7 3,8 3
      3 8 5,0 1 { Breadth inverted.
     -----------
  9)  5 2
     -----------
        5 8
      1 3 9 0
      8 6 9 1
    1 7 3 8 3 3   Sqrs.
100) 1 8 3 9,7 2 (1 8,3 9 7 3
```

In Roofing, Tyling and Slating, it is cuſtomary to reckon the flat and half of any Building within the Walls to be the Meaſure of the Roof of that Building, when the ſaid Roof is of a true Pitch.

Note, All Roofs are ſaid to be of a true Pitch, when the Rafters are $\frac{3}{4}$ of the Breadth of the Building.

If the Roof is more or leſs than true Pitch, they meaſure from one ſide to the other with a Rod or String.

QUESTION XIII.

If a House measure within the Walls 52 Feet, 8 Inches in Length, and 30 Feet, 6 Inches in Breadth; and supposing the Roof to be of a true Pitch; what will it cost Roofing at 10 *s.* 6 *d. per Square?*

By *Duodecimals.*

2) 30 — 6 — The Breadth of the Building
15 — 3 — The half Breadth
45 — 9 — The Breadth of the Roof
52 — 8

30 — 6 — 0
39 — 0
90
225

100) 2409 — 6

s.	d.		Sqr. F. I.	s. d.			
10		$\frac{1}{2}$	24 - 9 - 6 at 10 - 6 *per Square.*				
	6	$\frac{1}{20}$	12 - 0	F. I.		s. d.	
			— 12	5 and 4	$\frac{1}{20}, \frac{1}{25}$	10 - 6	
			— — - 11$\frac{3}{4}$			— 6$\frac{1}{4}$	
			12 - 12 - 11$\frac{3}{4}$	6	$\frac{1}{8}$	— 5	
						— $\frac{1}{2}$	
						— 11$\frac{3}{4}$	

By *Decimals.*

2) 30,5
15,25
45,75
52,6
9) 27450
30500
9150
22875
100) 2409,5
24,095 *Sqrs.*
,525
120475
48190
120475
12,649875

Answ. 12 *l.* 12 *s.* 11$\frac{3}{4}$ *d.*

There

THERE are other Works about a Building done by the Carpenter, that are measured by the Foot Running Measure; as Cornices, Doors, and Cases, Windows-Frames, Lintels, Guttering, Sommers, Skirt Boards, &c.

IN the measuring of Roofing for Workmanship alone, they generally deduct for the Holes for Chimny Shafts and Skylights, if they are any thing considerable.

BUT measuring for Work and Materials, they commonly measure in all Skylights, Lutheren Lights, and Holes for the Chimney Shafts, for their Trouble and Waste of Stuff; excepting such Skylights as are very large, that is, exceed nine or ten Feet in Area.

QUESTION XIV.

WHAT will the Tyling a Barn cost me at 25 *s.* 6 *d. per Square*; the Length being 43 Feet, 10 Inches, and Breadth 27 Feet, 5 Inches on the Flat; the Eaves Boards projecting 16 Inches on each Side?

By

By *Duodecimals*. | By *Decimals*.

```
              2) 27 -  5 ——— The Breadth ——— 2) 2 7,4 1 6 6
                 13 -  8 -  6 — The half Breadth — 1 3,7 0 8 3
1 - 4 × 2 =       2 -  8 — The Eaves Boards Doubled — 2,6 6 6 6
                 ___________                       __________
                 43 -  9 -  6                      4 3,7 9 1 6
                      43 - 10 - 0                    3 8,3 4  Length inverted.
                 ______________                 _____________
                 36 -  5 - 11 - 0                9) 1 3 1 4
                  1 -  9 -  6       s.  d.         _______
                 32 -  3          25 - 6 = 1,275     1 4 6 0
                 192                                3 5 0 3 3
                172                                1 3 1 3 7 5
          _______________                        1 7 5 1 6 6 6          Sqrs.
          100) 1919 -  6 -  5                100) 1 9 1 9,5 3 4 (1 9,1 9 5 3 4
               _______________                                          5 7 2,1
               S.   F.   I.  P.    s.  d.                            __________
               19 - 19 -  6 - 5 at 25 - 6 per Square.                  1 9 1 9 5 3
                                                                       3 8 3 9 0
 s.   |     |          |  s.   F.  |          |  s.   d.               1 3 4 3 7
 5 d  | 1/4 | 19       |  10,  4   | 1/10, 1/25 | 25 -  6                  9 6 0
 6.   | 1/10| 4 - 15   |       5   |    1/2   |  2 -  6½             __________
              9 -  6   |       I.  |          |  1 -  3¼              2 4,4 7 4
              4 - 11½  |       6   |    1/8   |  1 -  0¼
 Answ. l. 24 - 09 - 05½                       |        1½
                                              |  4 - 11½
```

Note,

Note, WHEN the Ridge of a Roof does not run ſtrait but bends into an Angle, then the Tyles coming from the Ridge to the Eaves make an Angle alſo; and that Angle of the Roof which bends inwards, is called a *Valley*; and that Angle that bends outwards, is called a *Hip*; and in Tyling and Slating, it is common to add the Length of the Vallies (meaſured from the Ridge to the Eaves) to the Content in Feet; ſometimes the Hips are added.

IN Slating it is common to reckon the Breadth of the Roof 2 or 3 Inches broader than what it meaſures, becauſe the firſt Row is almoſt cover'd by the ſecond; this is done ſometimes when a Roof is tyled.

Note, SKYLIGHTS and Chimney Shafts are deducted, but they ſeldom deduct Luthern Lights (or Garret Windows on the Roof) for the covering ſuch they reckon equal to the Hole in the Roof.

Fourthly. BY the Rod, as, Bricklayers Work.

RULE.

DIVIDE the Area found in Fact, by the Square of 16 ½ Feet, or 272 (the odd Quarter being ſeldom minded in Practice) and the Quote is the Rods.

Note, BRICKLAYERS always compute or value their Work at the Rate of a Brick and an half thick; and if the Thickneſs of the Wall happen to be more or leſs than ſuch, it muſt be reduced to that Thickneſs as follows:

MULTIPLY the Area of the Wall by the Number of half Bricks the Thickneſs of the Wall is of; and divide the Product by 3 (becauſe 1½ Brick is 3 halves) and it gives the Area.

QUESTION

QUESTION XV.

How many ſquare Rods are there in a Wall 62½ Feet long, 14 Feet, 8 Inches high, and 2½ Bricks thick?

By *Duodecimals.*

```
          92 -  6
               14 - 8
          41 -  8 - 0
          875 - 0  R.    F.    I.
    272) 916 -  8 (3 - 100 - 8
                              5
               3) 16 - 231 - 4
      R.   F.   I.   P.  5 - 167 - 9 - 4
ANSW. 5 - 167 - 9 - 4
```

By *Decimals.*

```
          6 2,5
           1 4,6
     9) 3 7 5 0
         4 1 6 6
        8 7 5 0   Rods.
  272) 9 1 6,6 (3,3 7, &c.
                  5
           3) 1 6,8 5
                5,6 1 6, &c.
```

In measuring of Brickwork they are commonly very careful in Regard to their Allowances; for one Foot square in the Front is commonly worth Sixpence, so that any considerable Error would amount to too much to reject.

In great Buildings they often deduct all the Timbers laid in the Walls, but this is only when the Workmanship is very good; for in general it is allow'd in, because they reckon the Time they generally wait on the Carpenter, together with the Bedding those Timbers in Mortar, is equal to the Brickwork that would supply the Timbers Place.

QUESTION XVI.

Suppose the Side Walls of an House 28 Feet, 10 Inches in Length, and the Height of the Roof from the Ground 53 Feet, 8 Inches, and the Gable (or Triangular Part at Top) to rise 42 Course of Bricks (reckoning 4 Course to a Foot). Now 20 Feet high is $2\frac{1}{2}$ Bricks thick; 20 Feet more 2 Bricks thick; 15 Feet 8 Inches more at $1\frac{1}{2}$ Brick thick; and the Gable 1 Brick thick; what will the whole Work come to at 5 *l.* 16 *s.* *per Rod?*

$$\frac{42}{4} = 10{,}5$$

$\frac{42}{4} = 10,5$ the Height of the Gable half is 5,25 *Feet.*

```
   28,83        28,83         28,83
      20           20           15,0
  ------       ------       ---------
  576,6        576,6       9)17300
      5            4          1922
  ------       ------        43250
3)28833      3)23066       ---------
  961,1         768,8       4517,2
```

```
    28,83
     5,25
  -------
   14416
   57666
 1441666
 --------
  151,375
        2
 --------
3) 302,75
 --------
  100,916   at 1   ⎫
  451,722   at 1½  ⎬ Bricks thick.
  768,888   at 2   ⎪
  961,111   at 2½  ⎭
 --------
 2282,638
```

   F.   *l.*   *F.*   *l.*
As 272 : 5,8 : : 2282,638 : 48,671

 l.   *s.*   *d.*
ANSWER, 48 — 13 — $5\frac{1}{4}$.

In all Buildings the Thickneſs of the Walls generally decreaſe as they riſe; and it is uſual to ſet off half a Brick at each Decreaſe: The Thickneſs is commonly ſet off on the Inſide, and that in a Place where a Floor will come, ſo that the Set-off is thereby hid.

It is common to build from a Baſe 4 Courſe of Bricks high, and which projects two or three Inches on each Side of the Wall.

The different Thickneſſes are meaſured ſeparate, and reduced each to a Brick and an half thick, and then added together.

If you are to meaſure a Chimney ſtanding alone by itſelf, without any Party-Wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth. The Thickneſs muſt be the ſame the Jaumbs are off, provided that the Chimney be wrought upright from the Mantle-Tree to the Ceiling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-Tree, becauſe of the Gathering of the Breaſt and Wings to make Room for the Hearth in the next Story.

If the Chimney-Back be a Party-Wall, and the Wall be meaſured by itſelf, then you muſt meaſure the Depth of the two Jaumbs, and the Length of the Breaſt for a Length, and the Height of the Story the Breadth, at the ſame Thickneſs your Jaumbs were off.

When you meaſure Chymney Shafts, girt them with a Line round about the leaſt Place of them for the Length, and the Height ſhall be your Breadth; and if they be four Inch Work, then you muſt ſet down your Thickneſs at one Brick-work; but if they be wrought nine Inches thick, (as ſometimes they are when they ſtand high and alone

alone above the Roof) then you muſt account your Thickneſs one and an half Brick, in Conſideration of Wyth and Pargetting, and Trouble of Scaffolding.

It is cuſtomary in moſt Places to allow double Meaſure for Chimnies.

More Examples *to exerciſe the foregoing* Propositions.

N. B. The Areas of Parallelograms and Triangles being divided by one of their Dimenſions, will give the other Dimenſion.

QUESTION XVII.

What Difference is there between a Floor 48 Feet long, and 30 Feet broad; and two other, each of half the Dimenſions?

$48 \times 30 = 1440$; and $24 \times 15 \times 2 = 720$; but 720 is half of 1440.

Therefore any plain Figure, whoſe Dimenſions for Meaſuring are double to the Dimenſions of another like Figure, contains four times the Area.

QUESTION XVIII.

A Mahogony Plank is 26 Inches broad; and I want a Yard and an half in Area ſaw'd off; what Diſtance from the End muſt the Line be ſtruck?

$1\frac{1}{2}$ Yards Area $= 13\frac{1}{2}$ Feet Area; and 26 Inches $= 2,1\dot{6}$ Feet.

Then

THEN $\frac{1350}{2,16} = 6,23$ Feet, the Diſtance from the End the Line muſt be ſtruck.

QUESTION XIX.

A JOIST is $8\frac{1}{2}$ Inches deep, and $3\frac{1}{2}$ broad; I want a Scantling juſt as big again, that ſhall be $4\frac{3}{4}$ Inches broad; what will be the other Dimenſion?

$8,5 \times 3,5 \times 2 = 59,5$ and $\frac{59,5}{4,75} = 12,52$ Inches deep, the Anſwer.

QUESTION XX.

I HAVE a Girder 19 Inches by 13; but one that has but a Quarter of the Timber in it, ſo it be 10 Inches deep will ſerve my Purpoſe; how broad muſt it be?

$19 \times 13 = 247$, and $\frac{247}{4} = 61,75$. Then $\frac{61,75}{10} = 6,175$ Inches, the Breadth.

QUESTION XXI.

I HAVE a Roof 24 Feet, 8 Inches, by 14 6 Inches on the Flat; and I would have it covered with Lead at 8 lb to the Foot; what will it come to at 18 *s. per Cwt.*

24,$\not{6}$ × 14,5 = 357,$\not{6}$ *F.* the Area, and 8 ℔ =,07232 *Cwt* ; as 1 *F.* : ,07232 *Cwt.* : : 357,$\not{6}$ *F.* : 25,866553 *Cwt.* the Weight.

Then as 1 *Cwt.* : ,9 *l.* : : 25,866453 *Cwt.* : 23,2798 *l.* &c. = 23 *l.* 5 *s.* 7 *d.* the Cost.

QUESTION XXII.

Suppose a Mason and a Plumber should agree as follows:

The Plumber to make the Mason a leaden Cistern, every Foot Square whereof should weigh 18 ℔. at 19 *s. per Cwt.* and the Dimensions to be 81 Inches long, 46 Inches deep, and 36 Inches broad; with three Stays across it, of the same Strength, and each 18 Inches deep: And the Mason to pave with Purbeck Stone, at 7½ *per Foot*, a Square, that should just balance the Cost of the Cistern; what must the Side thereof be?

81 + 36 = 117, which × by 2 = 234 = the Length of the two Sides and two Ends; or the Girth of the Cistern; then 234 × 46 = 10764 Inches, the Area of the Sides and Ends: And 81 × 36 = 2916 Inches, the Area of the Bottom.

Again, 36, the Length of one Stay, × by 18 the Depth = 648 Inches, which × 3 = 1944 Inches, the Area of the three Stays. Then 10764 + 2916 + 1944 = 15624 Inches, the Area of the whole Cistern and Stays.

Note, 19 ℔ = 1696 *Cwt.* &c.

Then, as 144 *sq. In.* : ,1696 *Cwt.* : : 15624 *sq. In.* : 18,4016 *Cwt.* The shortest Way to work this Proportion is, divide the third Term by the first Term, gives 108,5, and × the second Term by

by this Quote, gives the fourth Term, which is the Weight of the Ciſtern.

THEN if 1 *Cwt.* : ,95 *l.* : : 18,4016 *Cwt.* 17,48152 *l.* = 17 *l.* 9 *s.* 7½ the Expence that the Ciſtern does amount to.

THEREFORE, as ,03125 *l.* : 1 *Ft.* : : 17,48152*l.* : 559,40864 *Ft.* the Area of the Square; and by extracting the Square Root, you will have 23,65 *Ft.* &c. the Side thereof.

QUESTION XXIII.

I HAVE a Fleight of Iron Railing 42 Feet long, the Barrs whereof are ¾ of an Inch ſquare, and the whole Weight is 12¾ *Cwt.* But I have a mind to change them for ſome that are Inch and ⅘ ſtrong, and muſt give in Exchange 4½ *d. per* ℔; what will the whole come to?

IN order to ſolve this Queſtion, ſay by the *double Rule of Three:*

IF 42 Feet of ¾ Inches Strength weigh 12¾ *Cwt.* what will 42 of 1⅘ weigh?

Or, *in Decimals.*

F.	*I.*	*Cwt.*
42 ———	,75 ———	12,75
42 ———	1,8 ———	?

BUT the Strength of the Barrs are expreſſed by the Areas of the Ends; therefore inſtead of the Sides of the Barrs, put the Squares, or Areas at the Ends, and then it will be,

42 ———	,5625 ———	12,75
42 ———	3,24 ———	———

Here the Blank falls under the third Place.

AND

AND thus you ſhould proceed, ſuppoſing there were two different Lengths of Railing concern'd; but as the Length of the new is to be the ſame with the old, therefore this is ſolvable by the ſingle Rule, thus:

Strength. Cwt. Strength. Cwt.
As, ,5625 : 11,75 : : 3,24 : 73,44

℔ *d.* ℔ *l. s.*
IF 1 : $4\frac{1}{2}$: : 112 : 2 - 2

THEREFORE,

Cwt. l. Cwt. l. l. s. d.
As 1 : 2,1 : : 73,44 : 154,224 = 154 - 4 - $5\frac{1}{4}$

QUESTION XXIV.

A TRIANGULAR Field that is 1777,7 Links in the Baſe, and 900 Links in the Perpendicular, brings in 36 *l. per Annum*; how much is it let for *per Acre?*

2) 900 (450; then 1777,7 × 450 = 800000 *ſq. Links* which divide by 100000 (the ſquare Links in one Acre) gives 8 Acres; then 8) 36 *l.* (4,5 *l.* = 4*l.* 10 *s.* the Rent *per Acre.*

I BELIEVE the foregoing Examples are ſufficient to illuſtrate the Uſes of the firſt and ſecond Proportions; as alſo to make the Buſineſs of Decimals and Duodecimals very eaſy and applicable; in which I ſuppoſe the Reader ſo well acquainted by this time, that I ſhall not give the Solution of any of the ſucceeding Queſtions in the Manner I have done thoſe at the Beginning; but only ſhew how the Queſtion is to be wrought, and give the principal Reſults of the Proceſs, which I ſhall do Decimally, as being (in my Opinion) the moſt natural Way.

SECTION VII.

Of the Areas of divers Right-lined Figures.

PROPOSITION III.

The three Sides of a Triangle given to find the Area.

RULE.

From half the ſum of the three Sides, ſubtract each ſide ſeverally; then multiply the ſaid half Sum, with the three Differences continually; and out of this Product extract the ſquare Root; which Root will be the required Area.

QUESTION XXV.

A FIELD of a triangular Form hath it's Sides, 15, 14, and 13 Perches; what is the Area in ſquare Perches?

$15 + 14 + 13 = 42$; and $\frac{42}{2} = 21$, the half Sum of the three Sides, or $\frac{1}{2}$ Perimeter.

$21 - 15 = 6$, the firſt Difference.
$21 - 14 = 7$, the ſecond Difference.
$21 - 13 = 8$, the third Difference.

THEN $21 \times 8 \times 7 \times 6 = 7056$, whoſe ſquare Root is 84 ſquare Perches, the required Area.

QUES-

QUESTION XXVI.

A FIELD of a triangular Form whose Sides are 380, 420 and 765 Yards, lets for 55 *s.* *per Acre*; how much does the whole bring in *per Annum*?

$380 + 420 + 765 = 1565$, and $\frac{1565}{2} = 782,5$ Yards, the half Sum of the three Sides.

$782,5 - 380 = 402,5$, the first Difference.
$782,5 - 420 = 362,5$, the second Difference.
$782,5 - 765 = 17,5$, the third Difference.

$782,5 \times 402,5 \times 362,5 \times 17,5 = 1998003710,9375$, whose square Root is 44699,034 square Yards, &c. which divided by 4840 (the square Yards in one Acre) gives 9,2374 Acres.

THEN, as 1 *Acr.* : 2,75 *l.* : : 9,2374 *Acr.* : 25,40285 *l.* = 25 *l.* 8 *s.* $0\frac{1}{2}$ *d.* the Rent of the whole *per Annum.*

PROPOSITION IV.

Two Sides of a right angled Triangle being given, to find the other Sides.

THIS Proposition admits of two Cases.

First CASE.

THE two Sides (or Legs) perpendicular to each other being given, to find the other Side, called the *Hipothenuse*. *Fig*. 25.

RULE.

SQUARE each Side, and add the Squares together, and out of their Sum extract the square Root; and it gives the Hipothenuse as required.

Second CASE.

IF the Hipothenuse and one of the Legs be given, to find the other Leg.

RULE.

FROM the Square of the Hipothenuse subtract the Square of the given Leg; and extract the square Root out of the Remainder, gives the Leg required.

QUESTION XXVII.

I WANT the Length of a Shoar, that strutting 14 Feet from the upright of a Building, may support a Jaumb of $20\frac{1}{2}$ Feet from the Ground?

$20,5 \times 20,5 = 420,25$, and $14 \times 14 = 196$, then $420,25 + 196 = 616,25$, whose square Root is 24,82 Feet, *&c.* the Length of the Shoar required.

QUESTION XXVIII.

A LINE of 380 Feet will reach from the Top of a Precipice that stands close by a Brook-side, to the

the opposite Bank: Now it is known, the Precipice is 128 Feet high, how broad must the Brook be?

380 × 380 = 144400, and 128 × 128 = 16384, then 144400 — 16384 = 128016, whose square Root is 357,79 Feet, *&c.* the Breadth of the Brook required.

QUESTION XXIX.

A LADDER $52\frac{1}{2}$ Feet long may be so placed in a Street, that it shall reach a Window 29 Feet from the Ground on one Side; and by turning the Ladder over without removing the Foot, it will touch a moulding 40 Feet from the Ground on the other Side; how broad is the Street?

First, 52,5 × 52,5 = 2756,25, and 29 × 29 = 841, then 2756,25 — 841 = 1915,25, whose square Root is 43,76 Feet, the Breadth between the Ladder and Building the first Situation.

Secondly, 40 × 40 = 1600, and 2756,25 — 1600 = 1156,25, whose square Root is 34 Feet, the Breadth between the Ladder and Building, the second Situation.

THEN 43,76 + 34 = 77,76 Feet, the Breadth of the Street required.

PROPOSITION V.

To find the Area of a Parallelogram not rectangular.

RULE.

FROM either of the greater Angles let fall a Perpendicular to the Side opposite that Angle;

 and

and multiply that Perpendicular by the Length of the Side it falls on, and it gives the Area. *Fig*. 26.

QUESTION XXX.

THERE is a piece of Meadow Ground in Form of a Rhomboides, whose longest Side is 728 Yards, and the shortest Distance between the longest Sides [that is, the Perpendicular] is 358 Yards; Now supposing a Man can mow down the Grass standing on a square Rod in a Minute; how long will he be mowing the aforesaid Meadow, supposing him to work 14 Hours *per Day*.

728 × 358 = 260624 *Yds.* = 8615,669 *Rods.* = 8615,669 *Min.* = 10 *D.* 3 *H.* 3 *M.*

PROPOSITION VI.

To find the Area of a Trapezium.

RULE.

DIVIDE it into two Triangles by drawing a Line to any two of the Angles that are opposite, which Line is called the Diagonal; and from each of the other Angles let fall a Perpendicular to the Diogonal.

THEN Multiply the Sum of the two Perpendiculars by half the Diagonal; or, the Diagonal by half the Sum of the Perpendiculars, and the Product will be the Area.

OR find the Area of each Triangle, and their Sum will be the Area of the Trapezium.

QUES-

QUESTION XXXI.

WHAT is the Area of a Trapezium, whose Diagonal is 34 Feet, 9 Inches, and the Sum of the Perpendiculars is 28 Feet, 6 Inches?

By *Duodicimals.*		By *Decimals.*
F. *I.*		*F.*
34 - 9	The Diagonal	34,75
14 - 3 ½	Sum of the Perpendicul.	14,25
8 - 8 - 3		17375
10 - 6		6950
476 - 0		48650
495 - 2 - 3	The Area	495,1875

QUESTION XXXII.

THERE is a Meadow in Form of a Trapezium, in which the Perpendiculars cannot be easily found; but the Diagonal is 538 Yards; which dividing it into two Triangles is a Side in each, in one Triangle the other Sides are 283 and 471, and in the other, the remaining Sides are 432 and 216; what is the Area?

$538 + 471 + 283 = 1292$, and $\frac{1292}{2} = 646.$

THEN $646 - 538 = 108$, first Difference.
$646 - 471 = 175$, second Difference.
$646 - 283 = 363$, third Difference.

AND $646 \times 108 \times 175 \times 363 = 4432012200$, whose Root is 66573,35 Yards.

AGAIN,

AGAIN, $538 + 432 + 216 = 1186$, and $\frac{1186}{2} = 593$.

THEN $593 - 538 = 55$, first Distance.
$593 - 432 = 161$, second Distance.
$593 - 216 = 377$, third Difference.

AND $593 \times 55 \times 161 \times 377 = 1979632655$, whose Root is 44493 Yards.

THEN, $66573{,}35 + 44493 = 111066{,}35$ *sq. yds.* $=$ 22 *Acr.* 3 *Rds.* 31 *sq. Pol.* the ANSWER.

PROPOSITION VII.

To find the Area of a regular Polygon.

First, FIND the Center of the Figure, which is done thus:

IF the Figure has an even Number of Sides, then from any one Angle draw a Line to the Angle directly opposite, and the Middle of this Line is the Centre required. *Fig.* 27.

IF the Figure hath an odd Number of Sides; then bisect any two of the Sides, and from the Points of bisection draw Lines to the Angles directly opposite thereto, and the Point where these two Lines cut each other is the Centre of the Figure, as required. *Fig.* 28.

THE Center being thus found, from it let fall a Perpendicular to any one of the Sides, and multiply this Perpendicular by half the Sum of the Sides, and the Product is the Area.

QUES-

QUESTION XXXIII.

WHAT is the Area of a regular Pentagon whoſe Side is 25 Yards, and the Perpendicular let fall from the Center to one of the Sides is 17,2 Yards?

$25 \times 5 = 125$, and $\frac{125}{2} = 62,5$ the half Sum of the Sides.

THEN, $62,5 \times 17,2 = 1075$ ſquare Yards, the Area.

BUT the Areas of regular Polygons is more expeditiouſly obtained by the Help of a Table conſtructed in the following Manner.

THE Side of any of the regular Polygons is ſuppoſed Unity or 1; then by a Trigonometrical Calculation, the Perpendicular is found, and from thence the Area by the foregoing Rule.

THUS having found the Area of any regular Polygon, whoſe Side is 1; the Area of another like Polygon, whoſe Side only is given may be found, without ſeeking after the Perpendicular; for as the Square of the Side of one Polygon is to it's Area, ſo is the Square of the Side of any other like Polygon, to it's Area.

BUT becauſe the following Table ſhews the Area of Polygons from 3 Sides to 12, and the Side in each being Unity or 1, whoſe Square is alſo but 1; therefore multiply the Square of the Side of any given Polygon, by the correſponding Tabular Number, and it gives the Area.

Number

Number of Sides.	*Names.*	*Multipliers.*
3	*Trigon*	,4330133
4	*Square*	1,0000000
5	*Pentagon*	1,7204777
6	*Hexagon*	2,5980762
7	*Heptagon*	3,6339597
8	*Octagon*	4,8284271
9	*Nonagon*	6,1818242
10	*Decagon*	7,6942088
11	*Undecagon*	9,3656399
12	*Duodecagon*	11,1961524

EXAMPLES.

WHAT is the Area of a regular Heptagon, whose Side is 12 Inches?

LOOK in the Table against the Name *Heptagon*, and under Multipliers you will find 3,633959, which multiplied by the Square of the given Side 12 produces 523,290096 the Area of a regular Heptagon whose Side is 12.

AFTER the same Manner may the Area of any other regular Polygon (whose Name is in this Table) be found.

PROPOSITION VIII.

To find the Area of irregular Figures.

RULE.

DIVIDE the Figure into as many Triangles as may be, by drawing Lines from any one Angle to all the other Angles; and the Sum of the Areas of the ſeveral Triangles will be the Area of the Figure. *Fig.* 29.

BUT the Lines (frequently) may be better diſpoſed, than by drawing them all from the ſame Angle, *viz.* by dividing the Figure into Trapezia, as many as may be, and leaving as few ſingle Triangles as poſſible; for the Area of a Trapezium is more expeditiouſly obtained, than the Areas of the two Triangles which compoſe the Trapezium, found ſeparately. *Fig.* 30.

Note. ALL right lined Figures may be divided into as many Triangles (without any of the dividing Lines cutting each other) as the Figures hath Sides, abating two.

QUESTION XXXIV.

SUPPOSE a Meadow an irregular Figure of 8 Sides, which is let for 34 Shillings *per Acre*; upon the Menſuration is divided into three Trapezia: In the firſt, the Diagonal is 4 Chains and 24 Links, and the Sum of the Perpendiculars are 3 Chains 67 Links; in the ſecond the Diagonal is

7 Chains

7 Chains 43 Links, and the Sum of the Perpendiculars are 5 Chains 38 Links; in the third the Diagonal is 6 Chains 78 Links, and the Sum of the Perpendiculars are 4 Chains 84 Links, what will the whole bring in *per Annum? Fig.* 30.

4 *C.* 24 *L.* = 424 *L.* and 3 *C.* 67 *L.* = 367 *L.* then $367 \times \frac{1}{2} 424 = 77804$ *L.* the Area of the first Trapezium.

7 *C.* 43 *L.* = 743 *L.* and 5 *C.* 38 *L.* = 538 *L.* then $743 \times \frac{538}{2} = 199867$ *L.* the Area of the Second.

AGAIN, 6 *C.* 78 *L.* = 678 *L.* and 4 *C.* 84 *L.* = 484 *L*, then $678 \times \frac{484}{2} = 164076$ *L.* the Area of the third.

THEN $77804 + 199867 + 164076 = 441747$ *L.* = 4,41747 *Acres*, which at 34 *s.* = 1,7 *l.* comes to 7,509699 *l.* = 7 *l.* 10 *s.* $2\frac{1}{4}$ *d.* the Answer.

SECTION VIII.

Of a CIRCLE *and it's* PARTS.

HAVING thus far treated of right lined Figures, I come next to the Propositions relating to a circle, of which there are many; but I ſhall content myſelf with mentioning only thoſe of which there are frequent Uſe in practical Meaſuring;

Measuring; in order to which, it will be necessary to say something concerning the Proportions made Use of hereafter about the Circle.

THE antient Mathematicians (and indeed some of the Moderns) made several Attempts to find Numbers that would accurately give the Proportion between the Diameter of a Circle and it's Circumference; but it was found impossible, and then concluded, it was to be done no other ways but by Approximation, that is, by Numbers whose Value continually approached nearer the Truth, without ever arriving exactly at it.

ARCHIMEDES found out a Proportion in whole Numbers, *viz.* of 7 to 22, that is, supposing the Diameter of a circle to be 7; he found the Circumference of the same Circle to be 22 nearly; for 22 is to much.

METIUS found out a Proportion of 113 to 355; that is, supposing the Diameter of a Circle to be 113, he found the Circumference of the same Circle 355 nearly; but it is also too great, tho' much nearer the Truth than *Archimedes*.

VAN CULEN made a greater Progress in this Affair than any before him; for he supposing the Diameter of a Circle to be 1, found (with prodigious Labour and Trouble) the Circumference be 3,141592653.5897932384.6264338327.950288 which was then thought so great a Work, and so curious a Performance, that the Numbers were cut on his Tomb-Stone in the Church-Yard of St *Peter*'s at *Leyden*.

BUT by Methods of which the Moderns are now possessed, the same thing (and many others of the like intricate Nature) may be perform'd with abundantly less Labour and Trouble; as is sufficiently shewn by Mr *Jones* in his *Synopsis Palmariorum Ma-*

thescos, where he gives Mr *Machin*'s Proportion of the Diameter to the Circumference true to 100 Places; that is, ſuppoſing (as *Van Culen* did) the Diameter to be 1, he found the Circumference to be 3,1415922653.5897932384.6264338327.9502884 197.1693993751.0582097494.4592307816.40628 62089.9862803482.5342117067.9 + true to 100 Places of Figures, and that with very little Time and Trouble; a notable Inſtance of the Superiority of the Methods uſed by the Moderns, over thoſe of the Antients.

VAN CULEN's Proportion being of all others the moſt ſimple, as in ſome Caſes ſaving the Trouble of Multiplication, and in others of Diviſion, I ſhall all along confine myſelf to it; and here obſerve, that it is thought accurate enough to uſe no more of thoſe Numbers than 3,1416; and thus increaſing the fourth Place in the Decimals by Unity, the Deficiency in Operation ariſing by omitting, ſo many Figures [as muſt be done, to make the Work any thing tolerably eaſy] are in a Manner ſupply'd.

It may be expected that I ſhould here ſhew how theſe Proportions were found; but as I write this chiefly for the Uſe of thoſe who may not have had the Opportunity or Inclination to learn either Geometry or Algebra, I forbear it, knowing that if I ſhould inſert it, they would not readily underſtand it; and thoſe who have ſpent ſome Time in the Study of Geometry and Algebra, doubtleſs have found it well done already in many Authors; and I think in none better than in the above-mention'd Book.

I shall now lay down ſome Theorems concerning Circles, which are already demonſtrated in *Archimedes*, *Euclid*, and others.

I. The

I. THE Area of every Circle is equal to the Area of a rectangled Triangle, one of whose Legs is a Line equal to the Radius, and the other Leg equal to the Circumference; and hence arises one Method for finding the Area of a Circle, *viz.*

MULTIPLY half the Circumference by half the Diameter, and the Product is the Area.

II. THE Diameters and Circumferences of all Circles are in a direct Proportion to each other, and so are their like Chords and Arches, as are also the Sides of their inscribed and circumscribed Figures; that is, as the Diameter of one Circle is to it's Circumference; so is the Diameter of any other Circle, to it's Circumference, *&c.*

III. THE Areas of all Circles are in the Proportion to each other, as are the Squares of their Diameters; or as the Squares of their Circumferences; and also as the Squares of the Sides of the inscribed and circumscribed Figures; that is, as the Square of the Diameter of one Circle to it's Area; so is the Square of the Diameter of any other Circle to it's Area; and as the Square of one Circumference is to it's Area, so is the Square of any other Circumference to it'sArea, *&c.*

FROM these three Theorems are deduced all the Proportions concerning a Circle used in practical Measuring.

THE following Table shews the useful Multipliers and Divisors calculated to shorten the Operations relating to Circles:

If the Diameter be 1,	*The Circumference will be* 3,1416.	*The Area* ,7854.	*The Side of a Square equal* ,8862.	*The Side of an inscribed Square* ,7071.
If the Circumference be 1,	*The Diameter will be* ,31831.	*The Area* ,07958.	*The Side of a Square equal* ,2821.	*The Side of the inscribed Square* ,2251.
If the Side of a Square be 1,	*The Diameter of the circumscribing Circle will be* 1,4142.	*Diameter of a Circle equal* 1,1284.	*Circumference of the Circumscribing Circle* 4,443.	*Circumference of the Circle equal* 3,545.
If the Area of a Circle be 1,	*The Square of the Diameter will be* 1,27324.	*Square of the Circumference* 12,5664.	*The Area of the inscribed Square* ,6366.	

THIS

THIS Table will be sufficiently explained from the four following Propositions :

PROPOSITION IX.

The Diameter and Circumference of a Circle Being given to find the Area, and the Side of a Square whose Area shall be equal to that of the Circle or Square equal, and the Side of the Square inscribed in the Circle.

EXAMPLE

THE Diameter 1 and the Circumference 3,1416, what are the Rest ?

First, $\frac{3,1416}{2} \times \frac{1}{2} = 1,5708 \times ,5 = ,7854$ the Area *per* THEOR. I.

Secondly, THE square Root of ,7854 is ,8862 the Side of the square Equal *per* THEOR. III.

AND because the Side of the inscribed Square is the Hipothenuse of a right angled (*fig.* 31) Triangle whereof the Radii are the Legs : Therefore, the Radius $,5 \times ,5 = ,25$, which $\times\ 2 = ,5$, whose square Root is ,7071 the Side of the inscribed Square, *per* THEOR. III.

PROPOSITION X.

The Circumference of a Circle being given to find first, *the Diameter*; secondly, *the Area*; thirdly, *the Side of a Square Equal*; and fourthly, *the Side of the inscribed Square.*

EXAMPLE.

THE Circumference 1, what are the Rest?

First, 3,1416 : 1 : : 1 : ,318309 the Diameter, *per* THEOR. II.

Secondly, $\frac{,318309}{2} \times \frac{1}{2} = ,1591545 \times ,5 = ,07957725$, but ,07958 will do, which is the Area, *per* THEOR. I.

Thirdly, THE square Root of ,07958 is ,2821 the Side of the square Equal *per* THEOR. III.

Fourthly, $\frac{,318301}{2} = ,1591545$, then $,1591545 \times ,1591545 \times 2 = ,05068030974050$ whose square Root is ,225123, the Side of the inscribed Square, *per* THEOR. III.

THE Reason why the Square of the Radius is multiply'd by 2 is the same as in the last Example; *viz.* because the Side of the inscribed Square is the Hipothenuse of a right-angled Triangle, whereof the two Legs are each the Radii of the Circle. See PROPOSIT. IV. *Fig.* 31.

PRO-

PROPOSITION XI.

The Side of a Square being given, to find the Diameter and Circumference of the circumscribing Circle, and also the Diameter and Circumference of the Circle equal.

EXAMPLE.

The Side of a Square 1, what are the Rest?

First, the Diagonal of the Square being equal to the Diameter of the circumscribing Circle, (*fig.* 31) and it being also the Hipothenuse of a right-angled Triangle, whereof each Side of the Square is a Leg; therefore *per* (Prop. IV.) $1 \times 1 \times 2 = 2$, whose square Root is 1,4142, the Diameter of the Circle circumscribing the Square, *per* Theor. III.

Secondly, 1 : 3,1416 : : 1,4142 : 4,443, the Circumference *per* Theor. II.

Thirdly, ,7854 : 1 : : 1 : 1,27324 and whose square Root is 1,1284 the Diameter of the Circle equal in Area to the Square, *per* Theor. III.

Fourthly, 1 : 3,1416 : : 1,12837 : 3,544887192 or 3,545 the Circumference of the Circle equal, *per* Theor. II.

PROPOSITION XII.

The Area of a Circle being given; to find the Squares of the Diameter and Circumference, and also the Side of the inscribed Square.

EXAMPLE.

THE Area 1, what are the Rest?

First, ,7854 : 1 : : 1 : 1,27324, the Square of the Diameter, *per* THEOR. III. And by the last Example the Circumference of a Circle, whose Area is 1, was found to be 3,544887192, which squared gives 12,5662252, *&c.* the Square of the Circumference.

Secondly, IN the last Example the Diameter of a Circle whose Area is 1, was found to be 1,12837, then $\frac{1,12837}{2}$ = ,56418 and ,56418 × ,56418 = ,636598, *&c.* or ,6366 the Area of the inscribed Square.

FOR the Square of half the Radius is (*per* PROPOS. IV.) equal to twice the Square of half the Side of the inscribed Square and every Square being equal to four times the Square of half the Side; therefore twice the Square of the Radius, is equal to the inscribed Square, *fig.* 32.

THE following Examples will shew the Application of the Numbers in the Table found in the four foregoing Propositions.

EXAMPLE

EXAMPLE I.

SUPPOSE a Circle whoſe Diameter is 12 Inches; what is the Diameter, Area, Side of the Square Equal, and Side of the inſcribed Square?

Dia. 1	*Circumf.* : 31416	*Dia.* : : 12	*Circumf.* : 37,6992	The Circumference.
ſq.D. 1	*Area.* : ,7854	*ſq.D.* : : 144	*Area.* : 113,0976	The Area.
Dia. 1	*S.S.E.* : ,8862	*Dia.* : : 12	*S.S.E.* : 10,6344	The Side of the Square Equal.
Dia. 1	*S.I.S.* : ,7071	*Dia.* : : 12	*S.I.S.* : 84852	The Side of the inſcrib'dSquare.

EXAMPLE II.

SUPPOSE the Circumference 12, required the ſame things as before?

C. 1	*D.* : ,31831	*C.* : : 12	*D.* : 3,81972	The Diameter.
S.C. 1	*A.* : ,07958	*S.C.* : : 144	*A.* : 11,45952	The Area.
C. 1	*S.S.E.* : ,2821	*C.* : : 12	*S.S.E.* : 3,3852	The Side of the ſquare Equal.
C. 1	*S.I.S.* : ,2251	*C.* : : 12	*S.I.S.* : 2,7012	The Side of the inſcrib'dSquare.

EXAMPLE III.

LET the Side of a Square be 12, what are the Diameter and Circumferences of the circumscribing Circle, and Circle Equal?

SS. D.C.C. S.S. D.C.C. } The Diameter of the
1 : 1,4142 : : 12 : 16,9704 } circumscribing Circle.

SS. C.C.C. S.S. C.C.C. } The Circumference of the
1 : 4,443 : : 12 : 53,316 } circumscribing Circle.

S.S. D.C.E. S.S. D.C.E. } The Diameter of the
1 : 1,128 : : 12 : 13,536 } Circle Equal.

S.S. C.C.E. S.S. C.C.E. } The Circumference of
1 : 3,545 : : 12 : 42,54 } the Circle Equal.

EXAMPLE IV.

LET the Area of a Circle be 12, what is the Diameter and Circumference thereof, and the Side of the inscribed Square?

A. S.D. A. S.D. } whose square Root is
1 : 1,27324 : : 12 : 15,27888 } 3,9087 the Diameter.

A. Sq.Circ. A. Sq.Circ. } whose sq. Root is 12,2798 the
1 : 12,5664 : : 12 : 150,7948 } Circumference.

A. Sq.I. A. Sq.I. } whose square Root is 2,7639
1 : ,6366 : : ,12 : 7,6392 } the Side of the inscrib'd Square.

IF the Diameter or Circumference, &c. was any other Number, the Method of Operation would be exactly the same.

PROPO-

PROPOSITION XIII.

To find the Area of a Semicircle or Quadrant.

RULE.

MULTIPLY half the Arch Line by the Radius, and it givis the Area.

EXAMPLE I.

WHAT is the Area of a Semicircle, whose Diameter is 20?

As, 1 : 3,1416 : : 20 : 62,832, the whole Circumference; the half is 31,416, the Arch Lines. Then $\frac{31,416}{2} \times \frac{20}{2} = 157,08 \times 10$ gives 157,08 the Area of the Semicircle.

EXAMPLE II.

SUPPOSE the Diameter of a Circle be 16, what is the Area of a Quadrant (or fourth Part) thereof?

As 1 : 3,1416 : : 16 : 50,2656, whole the Circumference; the fourth part is 12,5664 the Length of the Arch-Line of the Quadrant. Then $\frac{12,5664}{2} \times \frac{16}{2} = 6,2832 \times 8 = 50,2656$, the Quadrant's Area.

THE most expeditious Method is thus:

First, For a Semicircle multiply 3,1416 by a fourth of the given Diameter, and multiply this

this Product by half the given Diameter, and it gives the Area.

Secondly, For a Quadrant multiply 3,1416 by an eighth Part of the given Diameter, and this Product by half the given Diameter, and it gives the Area.

THE Truth of these two Rules is manifest from the foregoing Operations.

PROPOSITION XIV.

Having the Diameter of a Circle, and the Distance from the Center a Chord-Line is to be drawn, to find the Length of that Chord.

RULE.

FROM the Radius subtract the Distance from the Center the Chord-Line is to be drawn, and the Remainder is the versed Sine; then mutiply the Difference between the Diameter and the versed Sine, by the versed Sine, and out of the Product extract the square Root, gives the Length of half the Chord, which doubled gives the Chord as required. *Fig.* 33.

EXAMPLE.

SUPPOSE a Circle 21,2 in Diameter; what is the Length of the Chord-Line drawn 5,6 from the Center?

$\frac{21,2}{2} = 10,6$ the Radius; then $10,6 - 5,6 = 5$, the versed Sine, and $21,2 - 5 = 16,2$, the Difference; then $16,2 \times 5 = 81$, whose square Root

Root is 9 = half the Chord: Therefore the whole Chord is 18.

HENCE it will be eaſy to find the Diameter by having the Chord and verſed Sine given; for divide the Square of half the Chord by the verſed Sine, and to the Quotient add the verſed Sine gives the Diameter: For Example; ſuppoſe the Chord 18, and the verſed Sine 5, what is the Diameter?

$\frac{18}{2} = 9$, the half Chord, and $9 \times 9 = 81$;

then $\frac{81}{5} = 16,2 + 5 = 21,2$, the Diameter ſought.

IT will be very eaſy to find the Chord of half the Arch, for it will be the Hipothenuſe of a right angled Triangle, whereof the verſed Sine will be one Leg, and the half Chord the other Leg; wherefore extracting the ſquare Root out of the Sum of the Squares of the verſed Sine and half Chord, (*per* PROP. IV.) will give the Chord of half the Arch. *Fig.* 33.

PROPOSITION XV.

To find the Length of any Arch of a Circle, by having the Chord of that Arch and the Diameter of the Circle given.

RULE.

DIVIDE the Cube of the Chord by ſix times the Square of the Diameter, and to the Quotient add the Chord, gives the Length of the Arch, very near. But if you would have it more accurate, then divide three times the fifth Power of the Chord, by 40 times the fourth Power of the Diameter, and

this Quotient add to the former Sum, gives the Length of the Arch very exact.

THERE are two other Rules given in the Books of Menſuration, *viz.*

MULTIPLY the Chord of half the Segment by 8, and from the Product ſubtract the Chord of the whole Segment; this Remainder divide by 3, gives the Arch-Line.

OR thus: To the double Chord of half the Segment's Arch, add one third of the Difference between the Chord of the whole Segment, and the double Chord of half the Segment's Arch, gives the Length of the Arch-Line.

THE Arch is obtained by either of theſe laſt Rules very eaſily; ſuppoſing the Chord of half the Arch be given; but if that be firſt to be ſought, from the Chord and Diameter, it requires a tedious Operation.

EXAMPLE.

SUPPOSE the Diameter 40, and the Chord 30; what is the Length of the Arch?

THIS Example I ſhall work by each Rule.

Firſt, $30 \times 30 \times 30 = 27000$, the Cube of the Chord, $40 \times 40 \times 6 = 10600$; then $\frac{27000}{10600} = 2{,}547$, &c. and $30 + 2{,}547 = 32{,}547$, the Length of the Arch: But if more Exactneſs be required, then $30 \times 30 \times 30 \times 30 \times 30 \times 3 = 72900000$, and $40 \times 40 \times 40 \times 40 \times 40 = 102400000$; then $\frac{72900000}{102400000} = {,}711$, and $32{,}547 + {,}711 = 33{,}258$, the Length very near.

Secondly, BY the other Methods; but firſt to find the Chord of half the Arch.

If

If a Line be drawn from the Center to one End of the Chord, then it is plain, that if from the Square of that Radius be taken the Square of half the Chord, the ſquare Root of the Remainder will be that Part of the Diameter intercepted between the Center and the Chord, which ſubtracted from the Radius leaves the verſed Sine; then the Square of the verſed Sine, and the Square of the Semi-chord added together, the ſquare Root of their Sum is the Length of the Chord of half the Arch.

See the Operation. $\frac{40}{2} = 20$, the Radius, $20 \times 20 = 400$, and $\frac{30}{2} = 15$, the Semi-chord, $15 \times 15 = 205$; then $400 - 225 = 175$, whoſe ſquare Root is $13{,}2287$, and $20 - 132287 = 6{,}7713$, the verſed Sine; then $6{,}7713 \times 6{,}7713 = 45{,}85050369$, and $225 + 45{,}85050369 = 270{,}85050369$, whoſe ſquare Root is $16{,}4575$, the Chord of half the Arch; then *per Rule ſecond.*

$16{,}4575 \times 8 = 131{,}66$, and $131{,}66 - 30 = 101{,}66$, then $\frac{101{,}66}{3} = 33{,}88$, the Arch's Length.

Again, *per Rule third*, $16{,}4575 \times 2 = 32{,}915$, and $32{,}915 - 30 = 2{,}915$, and $\frac{2{,}915}{3} = {,}9713$, then $32{,}915 + 9713 = 33{,}8863$, the Length of the Arch.

If the Length of the Chord of half the Arch be taken mechanically, I think the ſecond Rule the moſt expeditious, and is accurate enough for Practice, for the Difference of the Reſult in all the Methods is but ſmall.

PROPOSITION XVI.

To find the Area of a Sector.

RULE.

FIND the Length of the Arch intercepted between the two Radii, then multiply half that Arch by the Radius, and the Product is the Area. *Fig.* 34.

EXAMPLE.

WHAT is the Area of a Sector, the Radius being 24,5 and the Length of the Arch 62,6?

$\frac{62,6}{2} = 31,3$; then $31,3 \times 24,5 = 766,85$, the Area.

IF the Length of the Arch was greater than a Semicircle; then find the whole Circumference, and alſo the Length of the Arch wanting to compleat the Circumference, which ſubtracted from the Circumference leaves the Length of the Arch in the given Sector, the Area of which find as before. *Fig.* 35.

PROPOSITION XVII.

To find the Area of a Segment of a Circle.

RULE.

FIRST find the Center of a Circle, from which draw a Radii to the Ends of the Chord, and find the Area of that Sector, from which ſubtract the Area of the Triangle contain'd under the Chord and the two Radii, and the Remainder is the Area of the Segment. *Fig.* 36.

EXAM-

EXAMPLE.

Suppose a Segment of a Circle has a Chord-Line 24 Inches, and the versed Sine 9 Inches; what is the Area?

$\frac{24}{2} = 12$, and $12 \times 12 = 144$, and $\frac{144}{9} = 16$, and $16 + 9 = 25$, the Diameter (*per* Propos. XIV.) then $\frac{25}{2} = 12{,}5$, the Radius. Again, the Semi-chord $12 \times 12 = 144$, and the versed Sine $9 \times 9 = 81$, then $144 + 81 = 225$, whose square Root is 15 the Chord of half the Segment (*per* Propos. XV.) then $15 \times 8 = 120$, and $120 - 24 = 96$, and $\frac{96}{3} = 32$, the Length of the Arch, half is 16; then 12,5 the Radius multiply'd by 16, gives $200 =$ the Area of the Sector.

Again, 12,5 (the Radius) — 9 (the versed Sine) leaves 3,5 the Height of the Triangle made by the Chord and Radii, then $\frac{24}{2} = 12 \times 35 = 42$, the Area of that Triangle; then $200 - 42 = 158$, the Area of the Segment.

The same may be done without finding the Length of the Arch, by the following Rule.

Multiply the Difference between the Radius and versed Sine by 8 times the Radius, and this Product subtract from 14 times the Square of the Radius, from this Difference subtract 6 times the Square of the Difference between the Radius and versed Sine; and this last Remainder divide by 9 times the Radius, added to 6 times the Difference between the Radius and versed Sine, and the Quo-

tient multiply by half the Length of the Chord, and the Product is the Area.

TAKE the foregoing Example in order to compare the Methods, *viz.*

THE Chord 24 and the versed Sine 9, what is the Area?

$\frac{24}{2} = 12$ the $\frac{1}{2}$ Chord, and $12 \times 12 = 144$, and $\frac{144}{9} = 16$, and $16 + 9 = 25$ and $\frac{25}{2} = 12,5$, the Radius, and $12,5 - 9 = 3,5$, the Difference between the Radius and versed Sine.

$12,5 \times 8 \times 3,5 = 350$, and $12,5 \times 12,5 \times 14 = 2187,5$, then $2187,5 - 350 = 1837,5$, and $3,5 \times 3,5 \times 6 = 73,5$, then $1837,5 - 73,5 = 1764$.

AGAIN, $12,5 \times 9 = 112,5$, and $3,5 \times 6 = 21$, then $112,5 + 21 = 133,5$, and $\frac{1764}{133,5} = 13,21348$, then $13,21348 \times 12 = 158,56176$, the Area; nearly the same as by the former Rule.

PROPOSITION XVIII.

Having the Diameter of a Circle given, to find the Diameter of another Circle, whose Area shall increase or decrease by a given Ratio.

RULE.

MULTIPLY the Square of the given Circle's Diameter by the intended Increase, and out of that Product extract the square Root, which Root will be the Diameter of the Circle required.

BUT if the required Circle is to be less than the given Circle, then instead of Multiplying, divide by the

the intended Decreaſe, and out of the Quotient extract the ſquare Root.

EXAMPLE.

How much muſt that Circle be in Diameter, whoſe Area is 12 times as much as a Circle of 25 Inches in Diameter?

$25 \times 25 = 625$, the ſquare of the given Diameter, which multiply'd by 12, the intended Increaſe gives 7500, whoſe ſquare Root is 86,6 the Diameter required; but had it been to be decreaſed 12 times, then $\frac{625}{12} = 52{,}0833$, whoſe ſquare Root is 7,237 the Diameter of a Circle, whoſe Area is a twelfth Part of the Area of that Circle whoſe Diameter is 25.

PROPOSITION IV.

To find the Area of compound Figures.

Figures are ſaid to be compound, when the Sides thereof conſiſt of both right and curved Lines, and how to find the Area of ſuch Figures; the Way is, to every Curve Side draw a Chord, and then the Figure [excluſive of the Segments contain'd under the Curve Line and Chord] will be reduced to a right-lined Figure, whoſe Area find as directed in Prop. VIII. to which, if the Area of the Segments (found as *per* Prop. XVII.) be added, the Sum will be the whole Area. *Fig.* 37.

EXAMPLE

EXAMPLE.

SUPPOSE a Close of five Sides, two whereof are the Arches of Circles; now the Chords being drawn, the Figure will be a five-sided right-lined Figure, which being (by Lines drawn) divided into a Trapezium and a Triangle; in the Trapezium the Diagonal is 7 Chains, and the Sum of the Perpendiculars $6\frac{1}{2}$ Chains; in the Triangle the Base is 4 Chains, and the Perpendicular 6 Chains; in one Segment the whole Chord is 6 Chains, and the Chord of half the Arch is $3\frac{1}{2}$ Chains; in the other Segmet the whole Chord is 4 Chains, and the Chord of half the Arch is $2\frac{1}{2}$ Chains; what is the Area of this Figure?

First, $\frac{7}{2} = 3,5$, then $3,5 \times 6,5 = 21,75$, the Area of the Trapezium; *per* PROP. VI.

Secondly, $\frac{6}{2} = 3$, then $3 \times 4 = 12$, the Area of the Triangle; *per* PROP. II.

Thirdly, In the first Segment the Cord of half the Arch is 3,5, and half the whole Chord is 3, then $3,5 \times 3,5 = 12,25$, and $3 \times 3 = 9$, and $12,25 - 9 = 3,25$, whose square Root is 1,8 the versed Sine, then $\frac{9}{1,8} = 5$, and $5 + 1,8 = 6,8$, and $\frac{6,8}{2} = 3,4$, the Radius, and $3,4 - 1,8 = 1,6$, the Difference between the versed Sine and Radius; then $3,4 \times 8 \times 1,6 = 43.52$, and $3,4 \times 3,4 \times 14 = 161,84$, and $161,84 - 43,52 = 118,32$, and

1,6

$1,6 \times 1,6 \times 6 = 15,36$, and $118,32 - 15,36 = 102,96$, and $3,4 \times 9 = 30,6$, and $1,6 \times 6 = 9,6$, and $30,6 + 9,6 = 40,2$, then $\frac{102,96}{40,2} = 2,56$, *&c.* the Area.

Fourthly, IN the second Segment the Chord of half the Arch is 2,5 and half the whole Chord is 2, then $2,5 \times 2,5 = 6,25$, and $2 \times 2 = 4$, and $6,25 - 4 = 2,25$, whose square Root is 1,5 the versed Sine, then $\frac{4}{1,5} = 2,\dot{6}$, and $2,\dot{6} + 1,5, = 4,1\dot{6}$, and $\frac{4,1\dot{6}}{2} = 2,08\dot{3}$, the Radius, and $2,08\dot{3} - 1,5 = ,58\dot{3}$, the Difference between the Radius and versed Sine, then $2,08\dot{3} \times 8 \times ,58\dot{3} = 9,72$, and $2,08\dot{3} \times 2,08\dot{3} \times 14 = 60,763\dot{8}$, and $60,763\dot{8} - 9,72 = 51,041\dot{6}$, and $,58\dot{3} \times ,58\dot{3} \times 6 = 2,041\dot{6}$, and $51,041\dot{6} - 2,041\dot{6} = 49$, and $2,08\dot{3} \times 9 = 18,75$, and $5,8\dot{3} \times 6 = 3,5$, and $18,75 + 3,5 = 22,25$, then $\frac{49}{22,25} = 2,2$ the Area, then $21,75 + 12 + 2,56 + 2,2 = 38,51$ square Chains, and 10 square Chains being 1 Acre, therefore the Area of the Close is 3 Acres, 8 square Chains, 8 square Poles, *&c.*

SECT.

SECTION IX.

Practical Questions.

IF what has been already ſaid be well conſider'd and underſtood, the Buſineſs of ſuperficial Meaſuring ſo far as a right Line and a Circle is concern'd, muſt needs be very eaſy and familiar, and therefore I think there needs no more be ſaid on this Subject; but before I cloſe this Part, I will inſert a few Queſtions to exerciſe the young Meaſurer in the Buſineſs already done.

QUESTION I.

A round Pillar 7 Inches over, is ſufficient to carry a certain Weight, of what Diameter is the Column that contains 10 times the Stone on the ſame Length?

$7 \times 7 \times 10 = 490$, whoſe ſquare Root is 22,135 Inches, the Diameter as required.

QUESTION II.

A Brewer hath a Ciſtern which is fill'd by three Pipes, each of 3 Inches bore in a certain Time, of what Diameter muſt the Bore of that Pipe

Pipe be that in the ſame time will throw him in $2\frac{1}{2}$ times as much Water?

THE Quantity of Water thrown in being as the Squares of the Diameters, therefore $3 \times 3 \times 3 \times 2{,}5 = 67{,}5$, whoſe ſquare Root is 8,215 Inches, &c. the Diameter ſought.

QUESTION III.

SUPPOSE a Yard of Cable 9 Inches in Compaſs weighs 22 ℔, what will a Fathom weigh of that 9 Inches in Diameter?

1 *Circumf.* : ,31831 *Diam.* : : 9 *Circumf.* : 2,86479 *Diam.* whoſe ſquare is 8,2066, &c. then 8,2066 : 22 ℔ : : 81 (9×9) : 217,14, which $\times$ by 2, gives 434,28, the Weight ſought.

QUESTION IV.

I WANT in a Garden a Circular Pond, that ſhall juſt take up half an Acre, how long muſt the Chord be that will ſtrike the Circle?

THE half Acre contains 2420 ſquare Yards, therefore 1 : 1,2732 : : 2420 : 3081,2408 ſquare of the Diameter, whoſe ſquare Root is 55,5 the Diameter, the half of which is 27,75 Yards, the Length of the Line ſought.

QUESTION V.

AGREED for an Oaken Curb to a round Well, 8 *d. per* Foot ſquare; the Breadth of the Curb is to be $7\frac{1}{4}$ Inches, and the Diameter of the Well within the Brick-Work is $3\frac{1}{2}$ Feet, what will be the Expence?

$3\frac{1}{2}$ *F.* = 42 *In.* then 42 × 42 × ,7854 = 1385,4456, the Area within the Curb.

42 + 7,25 + 7,25 = 56,5, then 56,5 × 56,5 × ,7854 = 2507,19315 the Area of a Circle equal the Area of the Well and Curb, then 2507,19315 — 1385,4456 = 1121,74755, the Area of the Curb; now 8 *d.* = ,03 *l.* therefore as 144 : ,03 :: 1121,74755 : ,259671 *l.* = 5 *s.* $2\frac{1}{4}$ *d.* the Expence ſought.

QUESTION VI.

I WOULD have in a Garden a circular Pond with a circular Iſland in the Middle thereof; the Diameter of the Pond muſt be 100 Yards, and the Circumference of the Iſland the ſame; what will the digging the Pond come to at 18 *d. per* Foot ſquare on the Surface?

100 × 100 × ,7854 = 7854 Yards, the Area of the Pond and Iſland; 100 × 100 × ,07958 = 795,8, the Area of the Iſland; then 7854 — 725,8 = 7058,2 Yards, the Area of the Pond = 63523,8 Feet; then 1 : ,075 *l.* :: 63523,8 : 4764,285 = 4764 *l.* 5 *s.* $8\frac{1}{2}$ *d.*

QUESTION

QUESTION VII.

SUPPOSE the Expence of paving a Semicircular Plot, at 2 *s.* 4 *d. per* Foot amounted to 10 *l.* what is the Diameter thereof?

2 *s.* 4 *d* = $,11\dot{6}$, and if ,116 *l.* 1 *F.* : : 10 *l.* $\dot{8}5,714\dot{2}$ the Semicircle's Area, and $\dot{8}5,714\dot{2} \times 2 = 1\dot{7}1,428\dot{5}$ the Circle's Area; then 1 : 1,2732 :: $1\dot{7}1,428\dot{5}$: 218,262857l3, whose square Root is 14,7734 the Diameter sought.

QUESTION VIII.

SUPPOSE St *James's* Square to be 180 Yards long and 150 Yards broad, in which there is an Octagonal Gravel Walk, one of whose Sides is (suppose) 28 Yards; what did the Paving the rest with Purbeck Stone come to at 3 *s.* 6 *d. per* Yard?

180 × 150 = 27000 Yards, the Area of the Square.

28 × 28 × 4,828427 = 3685,486768 *Yds.* the Area of the Octagon.

THEN 27000 *Yds.* — 3685,486768 = 23314,513232 *Yds.* what was paved; and 1 *Y.* : ,175 *l.* : : 23314,513232 *Yds.* : 4080,0398156 *l.* = 4080 *l.* 00 *s.* $9\frac{1}{2}$ *d.* the whole Cost.

QUESTION IX.

WHAT is the Area of the Segment of a Circle whose Diameter is 50 Inches; supposing the Section made 14 Inches from the Center?

$\frac{50}{2} = 25$, the Radius, and $25 - 14 = 11$, the versed Sine. Then $25 \times 25 = 625$, and $14 \times 14 = 196$, and $625 - 196 = 429$, whose square Root is 20,7, *&c.* and $20{,}7 \times 2 = 41{,}4$, the Length of the Chord, *per* PROP. XIV.

AGAIN $25 \times 8 \times 14 = 2800$, and $25 \times 25 \times 14 = 8750$, and $8750 - 2800 = 5950$, and $14 \times 14 \times 6 = 1176$, and $5950 - 1176 = 4774$, the Dividend.

AGAIN, $25 \times 9 = 225$, and $14 \times 6 = 84$, and $225 + 84 = 309$, the Divisor.

THEN, $\frac{4774}{309} = 28{,}317$, and $28{,}\dot{3}17 \times 20{,}7 = 586{,}1619$, the Area sought, *per* PROP. XVII.

QUESTION X.

THREE Men bought a circular Cheese, 14 Inches in Diameter, which cost them 7 *s.* 6 *d.* whereof *A* pays 1 *s.* 4 *d.* *B* 2 *s.* 10 *d.* and *C* 3 *s.* 4 *d.* now they agree that it shall be divided from the Center to the Circumference; that is, it should be cut into three Sectors, the Area of which should bear the same Proportion to each other, as the Price each Man paid; what Part of the Circumference will fall to each Man's Share, together with the Areas?

$14 \times 14 = 196$, and $1 : {,}7854 :: 196 : 153{,}8384$, the whole Area.

7 *s.* 6 *d.*

7 *s*. 6 *d*. = ,375 *l*. 1 *s*. 4 *d*. = ,06 *l*. 2 *s*. 10 *d*. = ,1416 *l*. 3 *s*. 4 *d*. = ,16 *l*.

,375 : 153,8384 : : ,06 : 27,349, *A*'s Area.
,375 : 153,8384 : : ,1416 : 58,1167, &c. *B*'s Share.
,375 : 153,8384 : : ,16 : 68,372, &c. *C*'s Area.

As 1 : 3,1416 : : 14 : 43,9824, the whole Circumference. Then 153,8384 : 43,9824 : : 27,349 : 7,819 Circumference of *A*'s. 153,8384 : 43,9824 : : 58,1167 : 16,620 Circumference of *B*'s. 153,8384 : 43,9824 : : 68,372 : 19,547 Circumference of *C*'s. But the Circumference of each Man's Part may more easily be found, thus: as ,375 : 43,9824 : : ,06 : 7,819, *A*'s Part of the Circumference, ,375 : 43,9824 : : ,1416 : 16,62, *B*'s Part, ,375 : 43,9824 : : ,16 : 19,547, *C*'s Part of the Circumf.

QUESTION XI.

A, *B*, and *C* bought a Grinding-Stone of 21 Inches in Diameter, each paying a third Part; what Part of the Diameter must each grind down?

THIS Question is answer'd by reckoning each Man to grind away one third of the Area.

21 × 21 × ,7854 = 346,3614, the Area, and $\frac{346,3614}{3}$ = 115,4538, each Man's Area, which taken from the whole, leaves 230,9076, the Area of the Circle remaining, when one Man has ground away his Share. Then 1 : 1,2732 : : 230,9076 : 293,99155632, whose square Root is 17,1461, the Diameter of two Mens Shares next the Center. Again, 1 : 1,2732 : : 115,4538 : 146,99577816, whose square Root is 12,1241, the Diameter of one Man's Share next the Center. Then 21 — 17,1461 = 3,8539, the first Man's Part; and 17,1461 — 12,1241 = 5,022, the second Man's Part; and the third 12,1241.

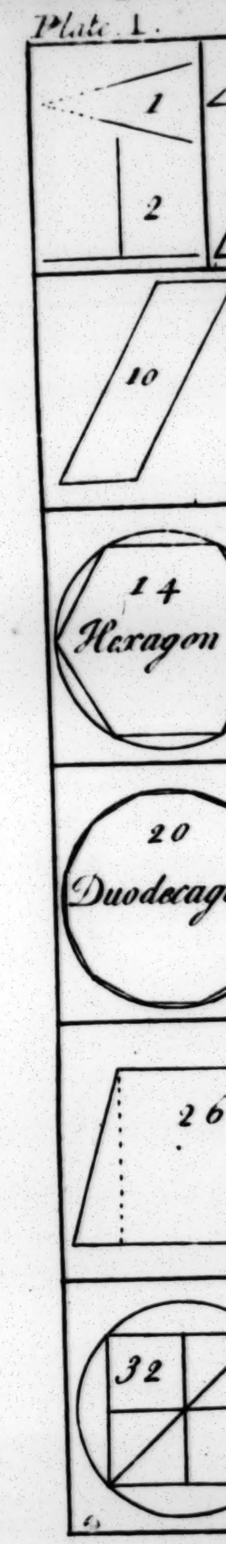
QUESTION

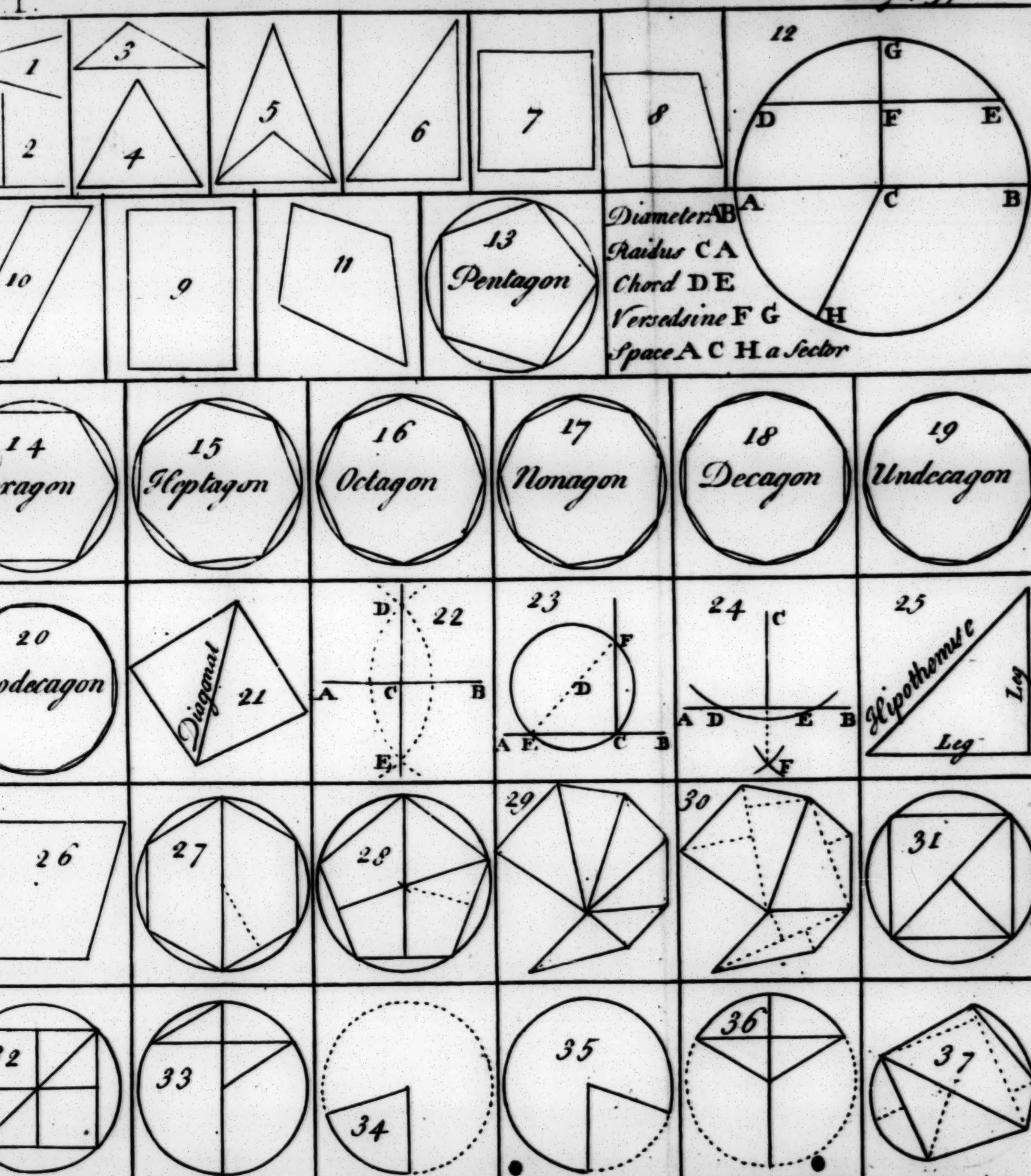
QUESTION XII.

ON a fair Country Green there once did ſtand,
A circ'lar Rail about a Piece of Land:
The greateſt Part o'the Rail is now quite gone,
And the Ground-Plot defac'd it ſtood upon;
But by what's left, I can find the Chord-Line,
And alſo get the Length o'the verſed Sine;
The Chord-Line's 40 *Yards the other's* 9.
Now I have order'd a new Rail be made,
Alſo a circ'lar Walk within be laid;
Whoſe Breadth's twelve Feet; the Price agreed is known;
For one Yards Length o'the Rail to be 1 *Crown;*
And for one Square Yards of the Gravel Way,
I to the Workman Eighteen Pence muſt pay;
Now tell to me the whole Expence I pray?

$\frac{40}{2} = 20$, and $20 \times 20 = 400$, and $\frac{400}{9} = 44{,}4$, and $44{,}4 + 9 = 53{,}4$, the Diameter of the Rail, $1 : 3{,}1416 :: {,}53{,}4 : 167{,}90106$, or $167{,}9$, the Circumference, or Length of the Rail; $\frac{53{,}4}{2} \times \frac{167{,}9}{2}$, or $26{,}72 \times 83{,}95 = 2243{,}3305$, Area of the outward Circle. 12 Feet = 4 Yards, and $4 \times 2 = 8$, the Walk's Breadth doubled. $53{,}4 - 8 = 45{,}4$, the Diameter of the inner Circle of the Walk, and $45{,}4 \times 45{,}4 \times {,}7854 = 1621{,}997131$, the Area of the inner Circle, and $2243{,}3305 - 1621{,}997131 = 621{,}333424$, the Walk's Area, 5 *s.* = ,25 *l.* then $167{,}9 \times {,}25$ *l.* = 41,974 *l.* = 41 *l.* 19 *s.* $5\frac{3}{4}$ *d.* the Price of the Rail. 18 *d.* = ,075 *l.* then $621{,}333424 \times {,}075 = 46{,}60000688$ = 46 *l.* 12 *s.* the Walk's Price; therefore $41{,}974 + 46{,}6 = 88{,}574$ *l.* = 88 *l.* 11 *s.* $5\frac{3}{4}$ *d*, the whole Expence.

OF

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OF

SOLID

MEASUREMENT.

PART II.

I SHALL *first* lay down the neceſſary Definitions; *ſecondly*, give the common Methods of meaſuring Timber; and *thirdly*, give the Methods whereby the Solidity of various Sorts of Figures are calculated.

SECTION I.

DEFINITIONS.

I. A SOLID is a Figure contained under three Dimenſions, *viz.* Length, Breadth, and Depth or Thickneſs.

II. A PRISM is a ſolid, whoſe Sides are Parallel (or more properly the Lines bounding the Sides are Parallel to each other) as are alſo the Ends.

III. A CUBE is a Priſm that hath equal Length, Breadth, and Depth [ſuch a Figure as a Die] and is contain'd under ſix equal ſquare Planes. *Plate* 2. *Fig.* 1.

IV. A PARALLELOPIPEDON is a Figure that hath it's Sides bounded or included within four equal Parallelograms, and two ſquare Baſes or Ends. *Fig.* 2.

V. A CYLINDER [or Solid like the Roling-Stone of a Garden] is only a round Priſm, having the Baſes or Ends perfect Circles. *Fig.* 3.

VI. A PYRAMID is a Solid, whoſe Baſe is a Polygon, and the Sides are Triangles, whoſe Tops all meet in a Point; which Point is called the Vertex of the Pyramid. *Fig.* 4.

VII. A CONE is a Solid ſtanding on a circular Baſe, and tapereth away to a Point called it's Vertex, in ſuch a Manner that all Lines from the ſaid Vertex along the ſlant Surface are ſtrait lines. *Fig.* 5.

VIII. A SPHERE or Globe is a Solid bounded within one regular Surface, and is formed by turning a Semicircle about it's Diameter. ——The Diameter of a Sphere is called it's *Axis*, and is equal to the Diameter of the generating Semicircle. *Fig.* 6.

IX. IF the ſmall End of a Pyramid or Cone be cut off, the Remainder is called a *Fruſtum*; and the Fruſtum of a Sphere is any Slice cut from the whole Sphere; and if a Fruſtum of a Pyramid or Cone be cut thro' diagonally, from the Extremity of one Side at the leſſer End, to the Extremity of the Baſe on the other Side, each of theſe Pieces is called a *Hoof*, or an *Ungula*, that being the greateſt which has the greateſt Baſe to it.

PROPO-

SECTION II.

PROPOSITION I.

To measure round Timber by the common Method.

RULE.

TAKE the Length of the Tree in Feet and Inches; then with a ſmall Cord or Chalk-line girt the Tree at an equal Diſtance from each End; one fourth Part of the Circumference is called by Artificers the *Girt*, and is (by them reckoned) the Side of a Square whoſe Area is equal to the Area of the Tree, ſuppoſing it to be cut through in the Place where it was girted; then multiply the Square of this Girt by the Length of the Tree, and it gives the Content.

WHAT is the ſolid Content of a Tree, whoſe Compaſs is 32 Inches, and the Length 9 Feet?

$\frac{32}{4} = 8$ Inches the Girt, and 8 Inches $= ,\dot{6}$ Foot.

$,\dot{6} \times ,\dot{6} = ,\dot{4}$, and $\dot{4} \times 9 = 4$ Feet, the Solidity.

EXAM-

EXAMPLE II.

WHAT is the solid Content of a Tree whose Length is 31 Feet, and the Girt 16 Inches?

16 Inches = 1,3̸ Feet. 1,3̸ × 1,3̸ × 31 = 54,07̸ Feet, the Solidity.

EXAMPLE III.

SUPPOSE a Piece of Timber 9¾ Feet long, and 39 Inches in the Girt; what is the Solidity?

39 Inches = 3,25 Feet. Then 3,25 × 3,25 × 9,75 = 102,984375, or 103 Feet, nearly.

EXAMPLE IV.

WHAT is the Solidity of a Tree whose Girt is 11 Inches, and the Length 40½ Feet?

11 Inches = ,916̸ Feet. Then ,916̸ × ,916̸ × 40,5 = 34,04475 Feet, or 34 Feet, nearly.

EXAMPLE V.

A TREE whose Girt is 31 Inches, and the Length 24 Feet; what is the Solidity?

31 Inches = 2,583̸ Feet. Then 2,583̸ × 2,583̸ × 24 = 160,16 Feet, the Solidity.

THOUGH this Method for it's Ease is commonly made Use of by Artificers, yet it is absolutely false; for the fourth Part of the Circumference of a Circle cannot be equal to the Side of a Square whose Area shall be equal to the

Area

Area of that Circle: For if the Circumference of a Circle be 1, the Side of a Square of equal Area to that Circle will be ,2821 ; whereas by the falſe Method of the Girt it is but ,25 ; therefore,

MULTIPLY the Compaſs of the Tree by ,2821, and it reduces it to the Side of a Square of equal Area with a Circle whoſe Circumference is equal to the Tree's Compaſs, which Side ſquare, and multiply the Square by the Length, and it gives the true Content, or Solidity.

TAKE the ſecond of the foregoing Examples, in order to compare the Difference of the Methods, *viz.*

THE Length 31 Feet, and the Girt 16 Inches; what is the Solidity?

16 × 4 = 64 Inches, which is 5,3 Feet, the Compaſs. Then 5,3 × ,2821 = 1,50453 Feet, the Side of a ſquare Equal. And 1,50453 × 1,50453 × 31 = 70,172, &c. the true Solidity. But by the common Way the Solidity is 55 Feet. Therefore 70,172 — 55 = 15,172 Feet, &c. a Difference too conſiderable to be neglected.

THE Solidity would be nearly the ſame, by ſaying as 11 : is to 14 : : ſo is the Solidity found by the common Way : to the true Solidity.

OR the Solidity may be thus found.

MULTIPLY the Square of the Compaſs by ,07958, and this Product by the Length, will give the Solidity required; for ,07958 is the Area of a Circle whoſe Circumference is 1.

TAKE Example the fifth, *viz.* A Tree 24 Feet long, and Girt 31 Inches; what is the Solidity?

BY the common Method the Solidity was found to be 160,16 Feet; then,

Firſt,

First, To find the true Solidity from the erroneous one.

11 : 14 : : 160,16 : 203,84, *&c.* The Solidity.

Secondly, By the Square Equal to the Tree's Compass.

31 × 4 = 124 Inches, and 124 Inches = 10,3 Feet, the Tree's Compass.

10,3 × 2821 = 2,91503, the Side of the Square Equal.

2,91503 × 2,91503 × 24 = 203,938064026, the Solidity.

Thirdly, By the Area of a Circle whose Circumference is equal to the Tree's Compass.

10,3 × 10,3 × ,07958 × 24 = 203,937013, the Solidity.

Here the Answer turns out almost 204 Feet, by these three different Methods; but I advise that either of the two latter Methods be used.

EXAMPLE VI.

Suppose the Length of a Tree to be 36,9 Feet, and the Compass 87 Inches; what is the Solidity?

First, By the Square Equal.

87 Inches = 7,25 Feet.

Then, 7,25 × ,2821 = 2,045225, the Side of the Square.

And 2,045225 × 2,045225 = 4,1829453, *&c.* the Square Equal.

Secondly, 7,25 × 7,25 × ,07958 = 4,18292375, the Circle Equal.

Now either of these multiply'd by the Length, gives the Solidity.

Thus

THUS the Circle equal, *viz.* $4,18292375 \times 36,75 = 153,7224478125$ Feet, the Solidity sought.

I THINK the Circle Equal in most Operations is found with less Trouble.

WHEN rough Timber (that is, Timber with the Bark on) is to be measur'd for Sale; there is generally some Allowance made for the Bark or Rind, which is one tenth Part of the Circumference; that is, subtract ,1 (one tenth) of the Tree's Compass, from the Compass, and it reduces it to such a Compass, as the Artificers (among whom this Method is used) suppose hath no Bark. But this Rule is only used in the Allowance for Oak-Bark: for the Allowance for Elm, Beach, Ash, *&c.* must be smaller.

IF a Tree have any Branches that measure two Feet in Compass, such Branches are called Timber and their Solidity is to be computed, and added to that of the Tree.

EXAMPLE VII.

THERE is a Tree, whose Length is 18 Feet, Compass $3\frac{1}{2}$ Feet, and hath two Boughs, the Length of one 10 Feet, and Compass $2\frac{1}{2}$ Feet, and the Length of the other is 12 Feet, and Compass 2 Feet, and being Oak, there is an Allowance to be made for Bark; what is the Solidity of the whole?

$$\frac{3,5}{10} = ,35$$

$\frac{3,5}{10} = ,35,$ and $3,5 - ,35 = 3,15.$

$\frac{2,5}{10} = ,25,$ and $2,5 - ,25 = 2,25.$

$\frac{2,0}{10} = ,2.$ and $,2 - ,2 = 1,8.$

So 3,15, 2,25, and 1,8 are the reſpective Compaſſes without Bark.

THEN, $3,15 \times 3,15 \times ,07958 \times 18 = 14,211,$ *&c.* the Tree's Solidity.

AND, $2,25 \times 2,25 \times ,07958 \times 10 = 4,027,$ *&c.* the firſt Bough's Solidity.

AND, $1,8 \times 1,8 \times ,07958 \times 12 = 3,094,$ *&c.* the ſecond Bough's Solidity.

THEN, $14,211 + 4,027 + 3,094 = 21,332$ Feet, the whole Solidity.

THE foregoing Method of girting the Tree at an equal Diſtance from each End, is only to be uſed when the Tree is nearly of the ſame Thickneſs in every Part ; but if it be very irregular, the Method then, is to gird it in ſeveral Places, (as many as is thought neceſſary) and the ſeveral Compaſſes together, and their Sum divide by the Number of Girts (which is thought by ſome) will give the mean Circumference.

Note, THE Buyer hath Liberty to gird the Tree any where between the Middle and the Ground-End if he chuſes it.

EXAMPLE

EXAMPLE.

SUPPOSE a Tree to be ſo irregular, that it is thought neceſſary to gird it in four ſeveral Places; the firſt Compaſs is 5,5 Feet; the ſecond is 4,7 Feet; the third is 3,6 Feet, and the Fourth is 4,2 Feet; the Length is 15 Feet; what is the Solidity thereof?

$$5{,}5 + 4{,}7 + 3{,}6 + 4{,}2 = 18, \text{ and } \frac{18}{4}$$

$= 4{,}5$ Feet, the mean Compaſs.

THEN, $4{,}5 \times 4{,}5 \times {,}07958 \times 15 = 24{,}173925$, the Solidity.

IF at any Time you would know how many Loads there are in any Quantity of Timber; find the ſolid Feet as before directed. Then if the Timber is hewn, divide the Feet by 50, gives the Loads; but if the Timber is unhewn, divide the Feet by 40, gives the Loads.

FOR the Meaſurers ſay that the Allowances given to 40 Feet of rough Timber, makes it equal to 50 Feet of hewn Timber, which is accounted to weigh a Ton or 20 hundred Weight, and ſo much is reckon'd a Cart Load.

WHAT has been already done, only concerns Timber, whoſe Ends are equal, or nearly ſo; but in the Books of Menſuration there are other Sorts of figur'd Timber conſidered; ſuch as unequal round Timber; ſquar'd,or unequal ſquare Timber the common Methods of calculating the Solidity of ſuch Timber is as follows:

Firſt, OF ſquared Timber; that is, ſuch Timber that when the Sides are hewn away, they are parallel, and the Areas of the Ends are equal; all ſuch Timber is meaſured thus:

FIND the Area of one End, which Area multiply by the Length of the Timber, and it gives the Solidity.

Secondly. OF unequal ſquared, and unequal round Timber; that is, ſuch Timber, that when hew'd into Form, have the Areas of their Ends unequal, that is, when the Dimenſions of the Figure at one End are different from thoſe of the other End.

THE common Methods of meaſuring ſuch Timber is; by finding a mean Girt as before directed.

BUT if ſuch Pieces of Timber are the Fruſtums of Pyramids or Cones, they had much better be meaſured as ſuch, by the Rule hereafter given; and therefore I think it needleſs to give any Examples, or ſay any thing more of them in this Place; for the Directions and Examples already given, take in all the Caſes that can poſſibly occur in the four Varieties, *viz*. Equal and Unequal, Squared, or Round; and otherwiſe than which, Meaſurers in their calculating the Solidity of Timber very ſeldom conſider.

SECTION III.

Of divers RIGHT-LINED *and* CIRCULAR FIGURES.

PROPOSITION II.

To find the Solidity of any Prifm.

RULE.

FIND the Area of the Bafe (or one End) which Area multiply by the Length, and it gives the Solidity.

EXAMPLE I.

WHAT is the Solidity of a Cube whofe Side is 12 Inches?

12 × 12 = 144, the Area of the End, and 144 × 12 = 1728, the Inches in a folid Foot.

EXAMPLE II.

WHAT is the Solidity of a Block of Marble, whofe Length is 10 Feet, Breadth $5\frac{3}{4}$ Feet, and Depth $3\frac{1}{2}$ Feet?

5,75 × 3,5 = 20,125, the Area of the End, and 20,125 × 10 = 201,25 Feet, the Solidity.

EXAMPLE III.

WHAT is the Solidity of a Triangular Prism, whose Length is 18 Feet, and one Side of the equilateral Base is $1\frac{1}{2}$ Feet?

IN the Table of PROPOS. VII. of *Superf. Meas.* the Area of an equilateral Triangle, whose Side is 1, is ,433014; therefore, $1,5 \times 1,5 \times ,433013 = ,97427925$, the Area of the End.

THEN, $,97427925 \times 18 = 17,5370265$, the Solidity.

EXAMPLE IV.

WHAT is the Solidity of a Cylinder, whose Length is 5 Feet, and the Diameter of the Base is 2 Feet? *Fig.* 3.

$2 \times 2 \times ,7854 = 3,1416$, the Area and $3,1416 \times 5 = 15,708$ Feet, the Solidity.

IT is customary in the Books of Mensuration, to give a Rule separate, for the working of each of the foregoing Examples; but I do not see the Necessity of it, as they all fall under one Definition, and may be wrought by one single Rule, *viz.* the foregoing one.

PRO-

PROPOSITION III.

To find the Solidity of any Pyramid or Cone. (Fig. 4, 5.)

RULE.

FIND the Area of the Baſe, which Area multiply by one third Part of the Altitude, and it gives the Solidity; becauſe every Pyramid or Cone is one third of a Priſm of the ſame Baſe and Altitude.

EXAMPLE I.

WHAT is the Solidity of a Pyramid, whoſe Height is 24 Feet, and the Side of it's ſquare Baſe is 3 Feet?

$3 \times 3 = 9$, the Area of the Baſe.

AND, $\frac{24}{3} = 8 = \frac{1}{3}$ of the Height.

THEN, $9 \times 8 = 72$, the Solidity ſought.

EXAMPLE II.

WHAT is the Solidity of a Pyramid whoſe Height is 15 Feet, and one Side of it's Hexagonal Baſe is 18 Inches?

IN the Table to PROP. VII. of *Superf. Meaſ.* the Area of a Hexagon, whoſe Side is 1, is found to be 2,598076.

THEN, $1{,}5 \times 1{,}5 \times 2{,}598076 = 5{,}845671$, the Area of the Baſe.

THEN, $5{,}845671 \times \frac{15}{3} = 29{,}228355$, the Solidity.

EXAMPLE III.

WHAT is the Solidity of a Cone, the Diameter of whose Base is 18 Inches, and the Altitude 15 Feet?

18 Inches $= 1,5$, and $1,5 \times 1,5 \times ,7854 = 1,76715$, the Area of the Base.

AND, $\frac{15}{3} = 5$, the Height.

THEN, $1,76715 \times 5 = 8,83575$ Feet, the Solidity.

PROPOSITION IV.

To find the Superficies of any right Cylinder.

RULE.

MULTIPLY the Circumference of the Base by the Length, and the Product is the Curve Superficies, to which add the Area of each End, and the Sum is the whole Superficies. For the Curve Superficies of every right Cylinder is equal to the Area of a Parallelogram, whose Length is equal to the Circumference, and the Breadth equal to the Height or Length of the Cylinder; for the Length multiply'd by the Breadth, gives the Area of the Parallelogram; and if this Parallelogram be circularly bent, till the two bounding End-Lines meet, it will form the Curve of the Cylinder.

EXAMPLE

EXAMPLE.

WHAT is the Superficies of a Cylinder whoſe Circumference is 100 Inches, and the Length 14 Feet?

100 Inches $= 8,3$ Feet, and $8,3 \times 14 = 116,6$ Feet, the Curve Superficies.

$8,3 \times 8,3 \times ,07958 \times 2 = 11,0537$, the Area's of the Ends.

THEN, $116,6 + 11,0537 = 127,7204$, the whole Superficies.

PROPOSITION V.

To find the Superficies of any right Cone.

RULE.

MULTILLY the Length of the ſlant Side by half the Circumference of the Baſe, and it gives the Curve Superficies; to which add the Area of the Baſe, and it gives the whole Superficies.

FOR the Curve Superficies of every right Cone is equal to the Area of a Sector of a Circle, whoſe Radius is equal to the ſlant Side of the Cone, and the Sector Arch, equal to the Circumference of the Cone's Baſe. For the Sector's Radius multiply'd by half it's Arch, will give it's Area; and if the Sector be circularly bent, till the Radii co-incide, it will form the Curve Superficies of the Cone.

EXAMPLE.

WHAT is the Superficies of a right Cone whose Circumference at the Base is 24 Feet, and the Length of the slant Side 32 Feet?

$32 \times \frac{24}{2} = 384$, the Curve Superficies.

AND, $24 \times 24 \times ,07958 = 45,93808$, the Area of the Base.

THEN, $384 + 45,93808 = 429,93808$, the whole Superficies.

PROPOSITION VI.

To find the Solidity of the Frustum of a Pyramid. (Fig. 7.)

RULE.

MULTIPLY the Sides of the greater and lesser Ends together, and to the Product add one third of the Square of the Difference of those Sides, which reduces the Ends to the Square of a mean Side; which Square multiply'd by it's proper Multiplier, taken out of the Table of Polygons Areas, *(Page* 106*)* will give the mean Area, which multiply by the Frustum's Length gives the Solidity.

EXAMPLE

EXAMPLE I.

What is the Solidity of the Fruſtum of a ſquare Pyramid, one Side of the greater End being 18 Inches, that of the leſſer End 15 Inches, and the Height 5 Feet?

18 Inches $=$ 1,5 Feet, and 15 In. $=$ 1,25 Feet.

Then, $1{,}5 - 1{,}25 = {,}25$, and $,25 \times ,25 = ,0625$, and $\frac{,0625}{3} = ,0208\dot{3} = \frac{1}{3}$ of the Square of the Side's Difference.

And, $1{,}5 \times 1{,}25 = 1{,}875$, and $1{,}875 + ,0208\dot{3} = 1{,}8958\dot{3}$, and $1{,}8958\dot{3} \times 5 = 9{,}4791\dot{6}$ Feet, the Solidity ſought.

EXAMPLE II.

What is the Solidity of a Hexagonal Pyramid's Fruſtum, the Side of whoſe ſquare End is 3 Feet, that of the leſſer End 2 Feet, and the Length 12 Feet?

$3 - 2 = 1$, and $1 \times 1 = 1$, and $\frac{1}{3} = ,\dot{3} = \frac{1}{3}$ of the Square of the Side's Difference.

And $3 \times 2 = 6$, and $6 + ,\dot{3} = 6{,}\dot{3}$, and $6{,}\dot{3} \times 2{,}598076$ (the Area of a Hexagon, whoſe Side is 1, *Page* 106) the Product is $16{,}454481\dot{3}$, the mean Area.

And $16{,}454481\dot{3} \times 12$ (the Length) gives 197,453776, the Solidity required.

If a Pyramid's Fruſtum's Ends be of any other Figure than that of a regular Polygon. Firſt find the Side of a Square, whoſe Area ſhall be equal to the Area of the greater End, and the ſame for the leſſer End; and then work as the foregoing Rule directs.

PRO-

PROPOSITION VII.

To find the Solidity of the Frustum of a Cone.

RULE.

To the Sum of the Squares of the Diameter of the End, add the Product of those Diameters; multiply this Sum by $\frac{1}{3}$ of the Height, and this Product by ,7854, and it gives the Solidity. *Fig.* 8.

Note. This Rule will serve for the Solidity of the Frustum of a Pyramid, omitting the Multiplication by ,7854.

And the Rule given for a Pyramid will serve for a Cone, by multiplying the Result given by that Rule by ,7854.

EXAMPLE.

What is the Solidity of the Frustum of a Cone, the Diameter of the greater End being 4 Feet, that of the lesser End 2 Feet, and the Altitude 9 Feet?

$4 \times 4 = 16$, and $2 \times 2 = 4$, and $16 + 4 = 20$, the Sum of the Squares of the Diameters.

$4 \times 2 = 8$, the Rectangle of the Diameters.

And, $\frac{9}{3} = 3 = \frac{1}{3}$ of the Height.

$20 + 8 = 28$, and $28 \times 3 \times ,7854 = 65,9736$ Feet the Solidity.

By these two last Propositions should all unequal'd squared or round Timber be measured; for most Trees

Trees being bigger at the Ground-End than at the other, may be conſidered (when firſt cut down, and the Branches lopt off) as the Fruſtums of Cones; and if the Sides are cut ſquare (or ſquariſh) they may then be conſidered as the Fruſtums of Pyramids, and conſequently in either Caſe, they ſhould be meaſured according to the Figure they repreſent, ſuppoſing them to be regular; but if the Difference of the Ends be ſmall, there is no need of having Recourſe to any other Directions than thoſe given in the firſt Propoſition; but as all Pyramids and Cones are conſidered as having their Sides perfectly ſtrait, moſt Trees will differ from them, by Reaſon of the Inequality of their Sides or Girths.

PROPOSITION VIII.

To find the Altitude of any Pyramid or Cone, having the Dimenſions of any Fruſtum of that Pyramid or Cone given.

RULE.

SAY, as the Difference between the two Diameters given (if a Cone) or between one Side of the greater End, and one Side of the leſſer End, (if a Pyramid) is to the Height of the Fruſtum, ſo is the greater Diameter (if a Cone) or one Side of the greater End (if a Pyramid) to the whole Height.

EXAMPLE.

THERE is a Fruſtum of a Cone, the Diameter of the greater End is 83 Inches, and that of the leſſer End is 54 Inches, and the Altitude is 12 Feet; what was the Altitude of the whole Cone ?

83 — 54 = 29 Inches, which is 2,416 Feet, the Difference of the Diameters, and 83 Inches = 6,916 Feet.

THEN, 2,416 : 12 : : 6,916 : 34,3448, &c. the Height of the whole Cone as required.

PROPOSITION IX.

To find what Length from the Vertex of a Pyramid or Cone will anſwer to any given Solidity

RULE.

SAY as the Solidity of the whole Pyramid or Cone, is to the Cube of it's Altitude, ſo is any given Part of the Solidity, to the Cube of it's Altitude reckon'd from the Vertex downwards, whoſe Cube-Root is the Length.

EXAM-

EXAMPLE.

I have a conical Piece of Timber, the Diameter of the Base is 18 Inches, and the Length 12 Feet; what Distance from the Vertex must I apply the Saw, to cut it into two Pieces of equal Solidity?

18 Inches $=$ 1,5 Feet. Then $1{,}5 \times 1{,}5 \times \frac{12}{3} \times {,}7854 = 7{,}0686$ Feet, the whole Solidity (*per* Prop. III.) half of which is 3,5343, and $12 \times 12 \times 12 = 1728$, the Cube of the Altitude.

Then, 7,0686 : 1728 : : 3,5343 : 864, the Cube of the Length of half the Solidity measured from the Vertex, whose Cube-Root is 9,5, &c. and so far from the Vertex must the Saw be apply'd to cut it into two equal Pieces.

PROPOSITION X.

To find the Solidity of the Hoofs or Ungulæ of the Frustum of a Pyramid. [Fig. 7.]

If it is of any other Form than a square Pyramid, it must first be reduced to such, by finding the Side of a Square of equal Area, Then

RULE.

To the Square of the Side of one End add one half, the Rectangle of the Sides of the two Ends, and this Sum multiply by one third of the Height gives the Solidity.

P. *Note.*

Note. YOU will have the Solidity of the greater or lesser Ungulæ, according to the End of the Frustum you use.

EXAMPLE.

THERE is the Frustum of a square Pyramid, one Side of the greater End is 1,5 Feet, the Side of the lesser End is 1,25 Feet, and the Height is 5 Feet; what is the Solidity of each Ungulæ?

$1,5 \times 1,5 = 2,25$, the Square of the greater Side.

AND, $1,5 \times 1,25 = 1,875$, and $\frac{1,875}{2} = ,9375$, and $2,25 + ,9375 = 3,1875$, and $\frac{5}{3} = 1,6̸$, then $3,1875 \times 1,6̸ = 5,3125$, the Solidity of the greater Hoof.

AGAIN, $1,25 \times 1,25 = 1,5625$, and $1,5625 + ,9375 = 2,5$, and $2,5 \times 1,6̸ = 4,16̸$, the Solidity of the lesser Ungulæ.

AND, $5,3125 + 4,16̸ = 9,47916̸$, the Solidity of the Frustum given.

IF this Example and the Result be compared with the Example and the Result in PROP. V. you will see how the Result of the Work agrees; for there the same Example is given to find the Solidity at once, which turns out the same as this does where it is found at two Operations.

PROPOSITION XI.

To find the Solidity of the Hoofs of a Cone's Frustum.

First, For the greater Hoof. *Fig.* 8.

RULE.

To the Square of the greater Diameter, add half the Product of the two Diameters, and to this Sum add the Difference of the Diameters; multiply this Sum by the Height, and this Product by ,2618, gives the Solidity.

Secondly, For the lesser Hoof.

RULE.

To the square of the lesser Diameter, add half the Product of the two Diameters; from this Sum subtract the Difference of the Diameters; multiply this Difference by the Height, and this Product by ,2618, gives the Solidity.

EXAMPLE.

THERE is a Cone's Fruſtum, the Diameter of the greater End is 4 Feet, that of the leſſer End 2 Feet, and the Height 9 Feet; what is the Solidity of each Hoof?

$4 \times 4 = 16$, and $4 \times 2 = 8$, and $\frac{8}{2} = 4$, and $4 - 2 = 2$, then $16 + 4 + 2 = 22$, and $22 \times 9 \times ,2618$ ($\frac{1}{12}$ of 3,1416) $= 51,8364$, the Solidity of the greater Hoof.

AGAIN, $2 \times 2 = 4$, and $4 \times 2 = 8$, and $\frac{8}{2} = 4$, and $4 - 2 = 2$, then $4 + 4 = 8$, and $8 - 2 = 6$, and $6 \times 9 \times ,2618 = 14,1372$, the Solidity of the leſſer Hoof.

AND, $51,8364 + 14,1372 = 65,9736$, the Solidity of the whole Fruſtum, which is exactly the ſame with the Solidity found by the Rule for finding the Solidity of a Fruſtum, which may be ſeen by comparing this with the Example to PROPOS. VI.

Note. There is a Miſtake in the Rule given in WARD's *Young Mathematician's Guide*; for the Multiplication ſhould be by the whole Height, and not by one third of the Height as there given.

SECT.

SECTION IV.

Of a Sphere *and it's Parts.*

PROPOSITION XII.

To find the Superficies of a Sphere or Globe. [*Fig.* 6.]

RULE.

MULTIPLY the Area of it's Circumscribing Circle by 4, and the Product is the Superficies.

EXAMPLE.

What is the Superficies of a Bomb-Shell, whose Diameter is 16 Inches?

16 Inches $= 1,\dot{3}$ Feet, then $1,\dot{3} \times 1,\dot{3} \times ,7854 = 1,3962\dot{6}$ Feet, the Area of the circumscribing Circle.

And, $1,3962\dot{6} \times 4 = 5,5850\dot{6}$, the Superficies.

Or thus: Multiply the Sphere's Axis into it's Circumference, gives the Superficies.

For, $1 : 3,1416 :: 1,\dot{3} : 4,1888$, and $4,1888 \times 1,\dot{3} = 5,5850\dot{6}$, as before.

PROPOSITION XIII.

To find the Solidity of a Sphere.

RULE.

FIND the Solidity of a Cylinder the Diameter of whoſe Baſe, as alſo it's Height are each equal to the Sphere's Axis, which Solidity multiply by 2, and the Product divide by 3, gives the Solidity of the Sphere; for every Sphere is equal to two thirds of it's circumſcribing Cylinder.

OR thus: If you ſuppoſe the Sphere to be made up of an innumerable Number of Cones, whoſe Vertices all meet at the Center, and the Areas of whoſe Baſes form the Superficies of the Sphere; now as the Solidity of any one Cone is equal to the Area of it's Baſe, multiply'd by one third of it's Height, therefore the Superficies of all the Cone's Baſes, multiply'd by the one third of the Height of one, gives the Solidity of all the Cones. But one third of the Height of one Cone is one ſixth of the Sphere's Axis, and the Superficies of the Baſes of all the Cones is equal to the Superficies of the Sphere, and the Solidity of all the Cones is equal to the Solidity of the Sphere; therefore,

RULE.

MULTIPLY the Superficies of the Sphere by one ſixth of the Axis, and the Product is the Solidity.

EXAMPLE

EXAMPLE.

WHAT is the Solidity of a Sphere, whose Diameter is 30 Inches?

30 Inches = 2,5 Feet, the Axis.

First. BY the first Rule. 2,5 × 2,5 × ,7854 = 4,90875, the Area of the Cylinder's Base.

AND, 4,90875 × 2,5 = 12,271875, the Cylinder's Solidity.

THEN, 12,271875 × 2 = 24,54375, and $\frac{24,54375}{3}$ = 8,18125 Feet, the Sphere's Solidity.

Secondly. BY the second Rule. 2,5 × 2,5 × ,7854 = 4,90875, the Area of the circumscribing Circle.

AND, 4,90875 × 4 = 19,635, the Sphere's Superficies.

AND, $\frac{2,5}{6}$ = ,416 = $\frac{1}{6}$ of the Axis.

THEN, 19,635 × ,416 = 8,18125, the Sphere's Solidity as before.

BUT, the Solidity of a Sphere may be more easily found, by finding the Solidity of a Sphere whose Diameter is 1, thus:

1 × 1 × ,7854 = ,7854, the Cylinder's Base.

AND, ,7854 × 1 (the Cylinder's Height) = ,7854, the Cylinder's Solidity.

THEN, ,7854 × 2 = 1,5708, and $\frac{1,5708}{3}$ = ,5236, the Sphere's Solidity.

THEN Spheres being in Proportion to each other as are the Cubes of their Diameters.

THEREFORE, As the Cube of 1 : is to ,5236, it's Solidity, so is the Cube of any other Sphere's Diameter to it's Solidity; that is,

RULE

RULE.

MULTIPLY the Cube of the Axis by ,5236, and the Product is the Solidity.

As in the foregoing Example where the Diameter is 30 Inches or 2,5 Feet. $2,5 \times 2,5 \times 2,5 \times ,5236 = 8,18125$ Feet, the Solidity, the same with the Solidity found by the two foregoing Rules.

IF by having the Circumference of a Sphere, you would find the Solidity; the best Way would be to find the Solidity of a Sphere, whose Circumference is 1; thus,

$3,1416 : 1 :: 1 : ,318309$, the Diameter.

AND, $\frac{,318309}{2} \times \frac{1}{2} = ,07958$, the Base's Area.

AND, $,318309 \times ,07958 = ,025331$, *&c.* the Solidity of the Cylinder.

AND, $\frac{,025331}{3} = ,0084436$, which $\times$ by 2 $= ,016887$, the Sphere's Solidity, whose Circumference is 1; then,

RULE.

MULTIPLY the Cube of the Circumference by ,016887, gives the Solidity.

PRO-

PROPOSITION XIV.

To find the Superficies of any Segment of a Sphere.

First, THE Axis and verſed Sine (or Height of the Segment) being given,

RULE.

SAY, as the Length of the Sphere's Axis is to the Sphere's Superficies, ſo is the verſed Sine to the Curve Superficies of the Segment, to which add the Area of it's Baſe, and it gives the Superficies of the whole Segment.

Secondly, BY knowing the Diameter of the Segment's Baſe, and the verſed Sine.

RULE.

To the Area of the Segment's Baſe, add the Area of a Circle whoſe Radius is equal to the verſed Sine, and the Sum is the Curve Superficies, to which add the Area of the Baſe, gives the whole Superficies.

EXAMPLE

EXAMPLE.

THERE is a Sphere whose Axis is 21 Inches; I want to know the Superficies of a Segment of this Sphere, cut off at 6 Inches from the Center?

OPERATION *by the first Rule.*

$\frac{21}{2} = 10,5$, and $10,5 - 6 = 4,5$, the versed Sine.

$21 \times 21 \times ,7854 \times 4 = 1385,4456$, the Superficies of the whole Sphere, *per* PROPOS. IX.

As $21 : 1385,4456 :: 4,5 : 296,8812$, the Curve Superficies of the Segment.

OPERATION *by the second Rule.*

BY the 14th *Prop. of Sup. Meas.* the Diameter of the Segment's Base will be 17,23368; then $17,23368 \times 17,23368 = 296,999$ *&c.* or 297 and $297 \times ,7854 = 233,2638$ the Area of the Segment's Base.

AND $4,5 \times 2 = 9$; then $9 \times 9 \times ,7854 = 63,6174$ the Area of a Circle whose Radius is the versed Sign.

THEN $233,2638 + 63,6174 = 296,8812$ the Curve Superficies of the Segment, the same as by the first Rule.

THEN $296,8812 + 233,2638 = 530,145$, the Superficies of the whole Segment, as was required.

PROPOSITION XV.

To find the Solidity of any Segment of a Sphere.
[*Fig.* 9.]

First, By having the versed Sign, and the Diameter of the Segment given,

RULE.

To three times the Square of half the Segment's Base add the Square of the versed Sign; multiply this Sum by the versed Sign, and this Product by ,5236, and the Product is the Solidity.

Secondly, By having the Axis, and versed Sine given.

RULE.

From three times the Axis subtract twice the versed Sine; multiply the Remainder by the Square of the versed Sine, and this Product multiply by ,5236, and the Product is the Solidity.

EXAMPLE I.

What is the Solidity of the Segment of a Sphere, the Diameter of the Segment's Base being 17,23368 Inches, and it's Height 4,5?

$\frac{17,23368}{2} = 8,61684$, and $8,61684 \times 8,61684$ $= 74,2499$, or $74,25$, and $74,25 \times 3 = 222,75$ = 3 times the Square of half the Segment's Base.

And, $4,5 \times 4,5 = 20,25$ = Square of the versed Sine.

Then, $222,75 + 20,25 = 243$, and $243 \times 4,5 \times ,5236 = 572,5566$, the Segment's Solidity.

EXAMPLE

EXAMPLE II.

THE Axis 21, what is the Solidity of the Segment of that Sphere, whose Height is 4,5?

$21 \times 3 = 63$, and $4,5 \times 2 = 9$, and $63 - 9 = 54$, the Difference between three times the Axis, and twice the versed Sine.

THEN, $4,5 \times 4,5 = 20,25$, and $20,25 \times 54 \times ,5326 = 872,5566$, the Segment's Solidity equal to that found by the former Rule.

THUS you may observe from the Examples to the four last Propositions, that the Result turns out the same (in those Examples where the same thing is sought after) although operated by different Rules.

PROPOSITION XVI.

To find the Solidity of the middle Frustum of a Sphere, otherwise called a Zone; *that is, a Sphere lessened by two Segments cut parallel to each other.* [Fig. 10.]

First. SUPPOSE the Zone cut through the Center parallel to the Ends, and call each Piece a Segment of the Zone; then will each Segment have two Ends, the Diameter of the greater being equal to the Sphere's Axis.

RULE.

TO twice the Area of the greater End, add the Area of the lesser End; and multiply the Sum by one third of the Distance of the Ends, gives the Solidity of that Segment of the Zone; or thus,

RULE.

FIND the Solidity of two Cylinders, the Height of each equal to the Height of the Segment, and the Diameter of ones Baſe equal to the greater End's Diameter of the Zone's Segment, and the Diameter of the others Baſe equal to the Diameter of the leſſer End of the Zone's Segment, ſubtract one third of the Difference of the Solidity of theſe Cylinders, from the Solidity of the greater Cylinder, and the remainder is the Solidity of that Segment of the Zone.

EXAMPLE.

SUPPOSE the Axis of a Sphere is 20 Inches; what is the Solidity of a Zone, the Section being made 6 Inches from the Center, and the Diameter at the Section being 16 Inches?

By the firſt Rule.

20 Inches $=$ $1,\not{6}$ Feet, and 16 Inches $=$ $1,\not{3}$ Feet, and 6 Inches $=$,5 Feet. $1,\not{6} \times 1,\not{6} \times ,7854 \times 2 = 4,34\not{3}$ $=$ twice the Area of the greater End.

$1,\not{3} \times 1,\not{3} \times ,7854 = 1,3962\not{6}$ $=$ leſſer End's Area.

AND, $\frac{,5}{3} = ,1\not{6} = \frac{1}{3}$ of the Height.

THEN, $4,34\not{3} + 1,3962\not{6} = 5,7396$, and $5,7396 \times ,1\not{6} = ,9566$, the Solidity.

By the second Rule.

$1,6 \times 1,6 \times ,7854 \times ,5 = 1,08583$, the greater Cylinder's Solidity.

AND $1,3 \times 1,3 \times ,7854 \times ,5 = ,69813 =$ lesser Cylinder's Solidity.

AND $1,08583 - ,69813 = ,3877$, and $\frac{3877}{3} = ,12923 = \frac{1}{3}$ the Difference of the Cylinder's Solidity.

THEN, $1,08583 - ,12923 = ,9566$, the Solidity as before.

SECTION V.

PRACTICAL QUESTIONS.

HERE follows a Collection of Questions contrived to exercise most of the foregoing Propositions, which (and many others of a like Nature) may be easily solved, if what has been delivered in the foregoing Pages be well understood.

QUESTION I.

How many 3 Inch Cubes connected together, will make a 12 Inch Cube? *Fig.* 11.

$\frac{12}{3} = 4 =$ Number of 3 Inch Cubes to make one Side of a 12 Inch Cube.

THEN, $4 \times 4 \times 4 = 64$, the Number of 3 Inch Cubes requisite to make a 12 Inch Cube.

QUESTION

QUESTION II.

A FARMER borrow'd of a Neighbour of his a Piece of a Haycock, which measured 6 Feet every way (that is a Cube whose Side was 6 Feet) and the borrowing Farmer paid back two equal cubical Pieces, each of whose Side were three Feet; Query whether the lending Farmer was fully paid?

$6 \times 6 \times 6 = 216$, the Solidity of the Piece borrowed.

$3 \times 3 \times 3 = 27$, and $27 \times 2 = 54$, the Solidity of the Pieces paid. Then,

$\frac{216}{54} = 4$, therefore the lending Farmer was paid but a fourth.

QUESTION III.

I WANT an Iron Roller for a Garden, whose outside Diameter shall be 20 Inches, Length of the Roller 50 Inches, and Thickness of the Metal $1\frac{1}{2}$ Inch; now supposing every Cube Inch weigh $4\frac{1}{4}$ Ounces, what will the whole come to at $3\frac{1}{4}$ *per* ℔?

$1\frac{1}{2} \times 2 = 3$, the double Thickness, and $20 - 3 = 17$, the Inner Diameter.

$20 \times 20 \times ,7854 = 314,16$, and $17 \times 17 \times ,7854 = 226,9806$, and $314,16 - 226,9806 = 87,1794$, and $87,1794 \times 50 = 4358,97$, the Solidity of the Roller.

$4\frac{1}{4}$ *oz.* $= ,265625$ ℔, and 1 *In.* : $,265625$ ℔ : : $4358,97$ *In.* : $1157,8514$ &c. the Weight.

$3\frac{1}{4}$ *d.* $= ,0135416$ *l.* and 1 ℔ : $,0135416$ *l.* : : $1157,8514 : 15,679$ *l.* &c. $= 15$ *l.* 13 *s.* 7 *d.* the whole Value of the Roller.

QUESTION IV.

I WANT two Pyramidal Lamp-Posts of Stone; I would have the Bases square, and each set on a Pedestal of 3 Feet in Height; the Height of the Pyramid must be 7 Feet, and the Side of it's Base 16 Inches, which is 2 Inches less than the Side of the Pedestal it stands on; what will they both come to at 2 *s.* 6 *d. per Foot? Fig.* 12.

16 Inches + 2 = 18 Inches = 1,5 Feet, one Side of the Base of the Pedestal.

AND, 16 Inches = $1,\cancel{3}$ Feet, $1,5 \times 1,5 \times 3 = 6,75$ Feet, the Solidity of one Pedestal.

AND, $1,\cancel{3} \times 1,\cancel{3} \times \frac{7}{3} = 4,0\cancel{7}$, the Solidity of one Pyramid.

THEN $6,75 + 4,0\cancel{7} = 10,77\cancel{7}$, the Solidity of one Post.

AND, $10,77\cancel{7} \times 2 = 21,54$, the Solidity of both.

THEN, 2 *s.* 6 *d.* = ,125 *l.* and 1 *F.* : ,125 *l.* : : $21,5\cancel{4}$ *F.* : $2,69\cancel{3}$ *l.* = 2 *l.* 13 *s.* $10\frac{1}{2}$ *d.* the whole Expence.

QUESTION V.

I HAVE given Orders for a Stone Cone to be made, to put in the midst of a Fountain in my Garden; the Height was to be 24 Feet, and the Diameter of the Base 12 Feet, and I have agreed with the Workman for 3 *s. per Foot solid*; but my Mind altering, I intend to have an Octagonal Pyramid of the same Height, and one Side of the Base to be 5 Feet; what will be the Difference of the Costs, supposing the Price to be the same in both?

$12 \times 12 \times ,7854 = 113,0976 =$ Area of the Base.

AND, $113,0976 \times \frac{24}{3} = 904,7808$ Feet, the Cone's Solidity.

THEN, 1 *F.* : 15 *l.* : : 904,7808 : 135,71712 *l.* the Cone's Expence.

$5 \times 5 \times 4,828427 = 120,780675 =$ Area of the Base.

AND, $120,71675 \times \frac{24}{3} = 965,6854$, the Pyramids Solidity.

THEN, 1 *F.* : ,15 *l.* : : 965,6854 *F.* : 144,85281 *l.* the Pyramid's Expence.

THEN, 144,85281 *l.* — 135,71712 *l.* = 9,13569 *l.* = 9 *l.* 2 *s.* 8½ *d.* the Difference of the Costs.

QUESTION VI.

WHAT will the Painting a conical Church Spire come to at 8 *d. per Yard*, supposing the Circumference of the Cone's Base to be 64 Feet and the Altitude 118 Feet?

1 : ,318309 : : 64 : 20,371776, the Diameter of the Base.

$\frac{20,371776}{2} = 10,185888$, the Radius.

AGAIN, $118 \times 118 = 13924$, and $10,185888 \times 10,185888 = 103,7523$, &c. and $13924 + 103,7523 = 14027,7523$, whose square Root is 118,43 Feet, the slant Side.

AND $118,43 \times \frac{64}{2} = 3789,76$ Feet, and $\frac{3789,76}{9} = 421,084$ Yards.

AND 1 *Yd.* : ,03 *l.* : : 421,084 *Yds.* : 14,03614 *l.* = 14 *l.* 0 *s.* 8¾ *d.* and so much will be the whole Expence.

QUESTION VII.

I HAVE in my Garden an Octagonal Pyramid, with a Cube fixed to it's Vertex; one Side of the Cube is equal to one Side of the Pyramid's Base, which is 9 Inches, and the Pyramid's Height is 7 Feet: I desire to know what the Guilding this Pyramid and Cube will come to, at 2 *d. per Inch?*

$9 \times 9 \times 6 = 486$, the Superficies of the Cube, and 7 Feet $= 84$ Inches.

IN order to find the Superficies of the Pyramid, the Length from the Vertex to the middle of the Sides must be found, which will be the Altitude of one Taiangle; but before the Length can be found, the Perpendicular Height of one of the Triangles composing the Base must be found. Thus,

$9 \times 9 \times 4{,}828427 = 391{,}102587$, the Area of the Base.

AND, $\frac{391{,}102587}{8} = 48{,}887823 =$ the Area of one of the Triangles composing the Base; the Base of which Triangle is the Side of the Polygon; therefore, $\frac{48{,}887823}{4{,}5} = 10{,}86396$, the Altitude of one of the Triangles composing the Base. Then the Height $84 \times 84 = 7056$, and $10{,}86396 \times 10{,}86396 = 117{,}025626$, *&c.* and $7056 + 117{,}025626 = 7173{,}025626$, *&c.* whose square Root is $84{,}69$, *&c.* the Height of one of the Triangles composing the Pyramid. Then $84{,}69 \times \frac{9 \times 8}{2} = 3048{,}84 =$ the Pyramid's Superficies. And $3048{,}84 + 468 = 3516{,}84$,

= 3516,84, the whole Superficies of the Pyramid and Cube. Then,

1 *In.* : ,008$\frac{1}{3}$ *l.* : : 35,1684 *In.* : 29,307 *l.* = 29 *l.* 6 *s.* 1$\frac{3}{4}$ *d.* the whole Expence.

QUESTION VIII.

I HAVE made a Marble Fruſtum of a Cone, for which I am to have 12 *s. per Foot* ſolid; the Diameter of the greater End is 4 Feet, that of the leſſer End 1,5 Feet, and the Length of the ſlant Side 8 Feet; what is the Fruſtum worth?

$\frac{4}{2} - \frac{1,5}{2} = 1,25$, and $8 \times 8 = 64$, and $1,25 \times 1,25 = 1,5625$, and $64 - 1,5625 = 62,4375$, whoſe ſquare Root is 7,9 Feet the Altitude of the Fruſtum.

THEN, $4 \times 4 = 16$, and $1,5 \times 1,5 = 2,25$; and $4 \times 1,5 = 6$, alſo, $\frac{7,9}{3} = 2,6\dot{3} = \frac{1}{3}$ the Altitude.

AND, $16 + 2,25 + 6 = 24,25$, and $24,25 \times 2,6\dot{3} \times ,7854 = 50,154335$ Feet, the Solidity of the Fruſtum.

AND, 1 *F.* : ,6 *l.* : : 50,154335 *F.* : 30,092601 *l.* = 30 *l.* 1 *s.* 10$\frac{1}{4}$ *d.* the Expence of the whole.

QUEST.

QUESTION IX.

SUPPOSE a Church-Spire was to be built of an octagonal Form, one Side of the greater End to be 24 Feet, one Side of the lesser End to be 12,5 Feet, and the Height 80 Feet; but the Inside of the Spire is to be run up in a conical Form, the Diameter of the Base is to be 56 Feet, and the Diameter at Top to be 28 Feet; I desire to know the Expence at 4 *s.* 6 *d. per Foot Solid?*

12,5 × 24 = 300, the Rectangle of the Sides of the Ends.

24 — 12,5 = 11,5 and 11,5 × 11,5 = 132,25, and $\frac{132\,25}{3}$ = 44,083 = to $\frac{1}{3}$ of the Square of the Difference of the two Sides.

AND, 44,083 × 4,828427 = 1661,381256916, the mean Area.

AND, 1661,381256916 × 80 = 132910,500553, or 132910,5 Feet, Pyramid's Frustum's Solidity.

56 × 28 = 1568, and 56 — 28 = 28, and 28 × 28 = 784, and $\frac{784}{3}$ = 261,3, and 1568 + 261,3 = 1829,3, and 1829,3 × 80 × ,7854 = 114930,672, the Solidity of the contain'd Cone.

THEN, 132910,5 — 114930,672 = 17979,828 Feet, the Solidity of the Stone Work.

THEN, 1 *F.* : ,225 *l.* : : 17979,828 *F.* : 4045,4613 *l.* = 4045 *l.* 9 *s.* 2¾ *d.* the Cost.

QUESTION X.

A MASON is employ'd to compleat a Cone, whose Vertex by Accident is struck off; he having made the upper Part level, the Dimensions of the Frustum are as follows: Length of the slant Side 12 Feet, Diameter of the upper End 6 Feet, and the Circumference of the Base 38 Feet; what will it cost at 3 *s. per Foot*, to put a Piece on, equal to that taken off?

As 1 : ,318309 :: 38 : 12,095742, or 12,1, the Diameter of the Base. 12,1 — 6 = 6,1, and $\frac{6,1}{2}$ = 3,05, the half Difference of the Diameters. Then 12 × 12 = 144, and 3,05 × 3,05 = 9,3025, and 144 — 9,3025 = 134,6975, whose square Root is 11,6, *&c.* the Frustum's Height. Then 6,1 : 11,6 :: 6 : 11,4, *&c.* the Height of the Piece wanting; and 6 × 6 × ,7854 × $\frac{11,4}{3}$ = 107,44272, *&c.* Feet, the Solidity of the Piece wanting. Then 1 *F.* : ,15 *l.* :: 107,44272, *F. &c.* : 16,1164, = 16 *l.* 2 *s.* 4 *d.* the whole Expence to compleat the Cone.

QUESTION XI.

THREE Men bought a tapering Piece of Timber, which was the Frustum of a square Pyramid; one Side of the greater End was 3 Feet, and one Side of the

the lesser End 1 Foot, and the Length 18 Feet; and they paying equally, are to have equal Shares; what is the Length of each Man's Piece? *Fig.* 13.

First, $3 \times 1 = 3$, and $3 - 1 = 2$, and $2 \times 2 = 4$, and $\frac{4}{3} = 1{,}3$, and $3 + 1{,}3 = 4{,}3$, and $4{,}3 \times 18 = 78$, the Solidity of the Frustum, and $\frac{78}{3} = 26$, the Solidity of each Man's Share. As $2 : 18 :: 1 : 9$, the Height of a Piece wanting; and $1 \times 1 \times \frac{9}{3} = 3$, the Solidity of the Piece wanting to compleat the Pyramid; then $26 + 3 = 29 =$ one Man's Solidity added to the Top, and $26 \times 2 = 52$, and $52 + 3 = 55 =$ two Mens Solidity added to the Top, and $78 + 3 = 81$, the Solidity of the whole Pyramid, and $18 + 9 = 27 =$ to it's Height; and $27 \times 27 \times 27 = 19683$, the Cube of the Pyramid's Altitude. Then $81 : 19683 :: 29 : 6923{,}543$, &c. whose Cube Root is 19,0; and $81 : 19683 :: 55 : 13365$, whose Cube Root is 23,7. Then $27 - 23{,}7 = 3{,}3$ *F.* the first Man's Length, and $23{,}7 - 19 = 4{,}7$ *F.* = second Man's Length, and $19 - 9 = 10$ *F.* = third Man's Length, accounting from the greater End for the first Man's, &c.

QUESTION XII.

THE Dimensions of a Cone's Frustum are as follows: Length of the slant Side 40 Inches, Diameter of the greater Base 50 Inches, and the Diameter of the lesser Base 20 Inches; if this Frustum was to be cut into two equal Pieces parallel to the Base; what would the gilding each Piece come to at 2 *d. per square Inch?*

$50 - 20 = 30$, and $\frac{30}{2} = 15$, and $40 \times 40 = 1600$, and $15 \times 15 = 225$, and $1600 - 225 = 1375$, whose square Root is 37,0, the Height of the Frustum. Then as $(30 - 20)$ 30 : 37 : : 20 : 24,6, the Height of the Piece wanting, and $37 + 24,6 = 61,6$, the whole Cone's Height. And

$$50 \times 50 \times \frac{61,6}{3} \times ,7854 = 40360,83,$$ the whole

Cone's Solidity. And $$20 \times 20 \times \frac{24,6}{3} \times ,7854$$ $= 2583,093$, the Solidity of the Piece wanting. And $40360,83 - 2583,093 = 37777,74$, the Solidity of the given Frustum. And $\frac{37777,74}{2}$ $= 18888,84$, and $18888,84 + 2583,093 = 21471,963$, the Solidity of half the Frustum added to the Solidity of the Piece wanting to make the Cone. And $61,6 \times 61,6 \times 61,6 = 234504,63$, the Cube of the whole Cone's Altitude. And 40360,83 : 234504,63 : : 21471,963 : 124756,454, &c. whose Cube Root is 49,9, &c. or 50 Inches, and $50 - 24,6 = 25,3$, the Length of half the Frustum's Solidity measur'd from the small End.

AGAIN, as the whole Cone's Height 61,6 : it's Diameter at the Base 50 : : the upper Frustum and Piece wanting, their Heights : to Diameter of the Frustum's Section.

As 37 : 40 : : 24,6 : 26,6, slant Side of the Piece wanting; and $40 + 26,6 = 66,6$, the whole Cone's slant Side; and 50 : 66,6 : : 40,5 : 54, the slant Side of the upper Frustum and Piece wanting. The Curve Superficies of the first Cone is 5236, and the Area of it's Base 1963,5; that of the second Cone 3435,3396, and it's Base's Area 1288,24235; that

that of the third Cone 837,76, it's Base's Area 314,16. Then 5236 — 2435,3396 = 1800,6604, and 1800,6604 + 1963,5 + 1288,24235 = 5052,40275, the whole Superficies of the lower Piece.

AND, 34353396 — 837,76 = 2597,5796, and 2597,5796+1288,24235+314,16=4199,98195, the whole Superficies of the upper Piece of the given Frustum. And 5052,40275 + 4199,98195 =9252,3847. Then 1 *In.* : ,008$\frac{1}{3}$ *l.* :: 9252,3847 : 77,1032, *&c.* = 77 *l.* 2 *s.* 0$\frac{3}{4}$ *d.* the whole Expence.

QUESTION XIII.

WHAT is the Weight of a Cannon-Ball of 7 Inches in Diameter supposing one of 4 Inches weighs 9 ℔?

4 × 4 × 4 = 64, and 7 × 7 × 7 = 343, and 64 : 9 ℔ : : 343 : 48,2 ℔, the Ball's Weight: For it will always be as the Cube of 4 is to it's Weight 9 ℔, so is the Cube of any other Ball's Diameter to it's Weight.

QUESTION XIV.

WHAT is the Diameter of a Ball that weighs 72 ℔?

As 9 ℔ : 64 : : 72 ℔ : 512, whose Cube Root is 8, the required Diameter: For it will always be, as 9 : 64, so is any Weight given to the Cube of the Diameter, whose Root will be the Diameter.

QUESTION

QUESTION XV.

SUPPOSE the Globe or Ball on the Top of St *Paul's* Church to be 6 *F.* in Diameter; what did the Gilding thereof come at $3\frac{1}{2}$ *d. per* Inch square.

1 : 3,1416 : : 72 *In.* : 226,1952 *In.* the Circumference, and 226,1952 × 72 = 16286,0544, the Superficies. And $3\frac{1}{2}$ = ,01458*3*. Then, 1 *In.* : ,01458*3* *l.* : : 226,1952 : 237,504 *l. &c.* = 237 *l.* 10 *s.* 1 *d.* the Expence.

QUESTION XVI.

WHAT is the Weight of a Bomb-Shell, that is 16 Inches Diameter without the Metal, which suppose is 3 Inches, admitting every Cubic Inch weighs $4\frac{1}{2}$ Ounces?

16 × 16 × 16 × ,5236 = 2144,6656, and $\overline{16 - 3 \times 2}$ = 10, and 10 × 10 × 10 × ,5236 = 523,6, and 2144,6656 — 523,6 = 1621,0656, and as 1 *C. I.* : ,28125 ℔ :: 1621,0656 : 465,9247 ℔ the Weight.

QUESTION XVII.

I HAVE a Cylindric Vessel the Depth is 28 Inches, and the Diameter of the Base is 46 Inches; but I want another that shall hold just as much again, whose Depth shall be 3 Feet; what must be the Diameter?

46 × 46 × ,7854 × 28 = 46533,3792, the Contents of the given Vessel. And 46533,3792 × 2 = 93066,7584, the Contents of the required Vessel. And $\frac{93066,7584}{36\ (3\ F.)}$ = 2585,1877, the Area of the required Vessel's Base.

1 : 1,2732 : : 2585,1877 : 3292,460725, whose square Root is 57,3797, the Diameter of the Vessel's Base, as required.

QUESTION XVIII.

I HAVE a Grainery 47 Feet 8 Inches long, 18 Feet 5 Inches broad, and 9 Feet 7 Inches high; but I want another that will hold four times as much, and have the Dimensions in the same Proportion to each other as the old one hath; what will be the Dimensions of the new one?

47 Feet 8 Inches = 47,6̸, and 18 Feet 5 Inches = 18,416̸, and 9 Feet 7 Inches = 9,583̸. Then 18,416̸ × 9,583̸ × 47,6̸ = 8412,83564̸81̸, the Solidity of the old Grainery. And 8412,83564̸81̸ × 4 = 33651,34259̸25̸, the Solidity of the new one. And 47,6̸ × 47,6̸ × 47,6̸ = 108303,96̸, the Cube of the Length of the old one.

8412,83564̸81̸ : 108303,96̸ : : 33651,34259̸25̸ : 433215866, &c. whose Cube Root is 75,6, &c. the Length of the new one.

47,6̸ : 75,6 : : 18,416̸ : 29,2, &c. the Breadth of the new one.

47,6̸ : 75,6 : : 9,583̸ : 15,2 nearly, the Depth or Height of the new one.

QUESTION XIX.

I HAVE a Vault to be dug, whose Length must be 20 Feet, Breadth 12 Feet, and Depth 7 Feet; and I am to pay the Workman 4 *s.* 6 *d. per* Floor for Work and Carriage; what will the Digging cost me?

12 × 7 × 20 = 1680 Feet, the Contents, and $\frac{1680}{324}$ = 5,1̸83̸ Floors. Then 1 *Fl.* : ,225 *l.* : : 5,1̸83̸ *Fl.* : 1,166625 *l.* = 1 *l.* 3 *s.* 4 *d.*

Note, 18 Feet square, and 1 Foot deep, or 324 Feet solid, is called a Floor.

QUESTION

QUESTION XX.

I REMEMBER once to have met with a Cone's Frustum, whose Solidity was 65,9736 Feet, and the Height was 9 Feet; the Diameter was in the Proportion of 2 to 1; what were those Diameters?

$\frac{65,9736}{9} = 7,3304$, the mean Area. As 1 : 1,2732 :: 7,3304 : 9,33306528, the Square of the Diameter of the mean Area. And $2 \times 1 = 2$, and $2 - 1 = 1$, and $1 \times 1 = 1$, and $\frac{1}{3} = ,\dot{3}$, and $2,\dot{3} =$ a mean Square between the Squares of the given proportional Numbers 2 and 1. Then,

As $2,\dot{3}$: 9,33306528 : : 4 (2×2) : 16, whose square Root is 4, the greater Diameter; and 2 : 1 : : 4 : 2, the lesser Diameter.

BUT all Questions of this Nature may be expeditiously solved by the following Rule. But first,

OF those two Numbers given in the Proportion as the Diameter sought, call the greater the *Antecedent*, and the lesser the *Consequent*. Then,

RULE.

MULTIPLY the given Solidity by the Square of the Antecedent, for a Dividend: To the Rectangle of the Antecednt and Consequent add the Square of each, and this Sum multiply by the Height,

and this Product by ,2618 ($\frac{1}{3}$ of ,7854) for a Divisor; then Division being made, the Quotient, will be the Square of the greater Diameter, whose Root will give the Diameter. Take the foregoing Example.

$2 \times 2 = 4$, and $65{,}9736 \times 4 = 263{,}8944$, the Dividend.

$2 \times 2 = 4$, and $1 \times 1 = 1$, and $2 \times 1 = 2$, and $4 + 1 + 2 = 7$, and $7 \times 9 \times {,}2618 = 16{,}4934$, the Divisor. Then, $\frac{263{,}8944}{16{,}4934} = 16$, whose square Root is 4, the greater Diameter, and $2 : 1 :: 4 : 2$, the lesser Diameter.

QUESTION XXI.

SUPPOSE two Porters having a Quart of strong Beer between them, agree to drink it off at two Pulls, that is, a Draught to each; now the first having given it the black Eye as they call it, that is, drank till the Surface of the Liquor touch'd the opposite Edge of the Bottom, he gave the remaining Part of it to the other; what was the Difference of their Shares? Supposing the Quart Pot was the Frustum of a Cone; the Depth being 5,7 Inches, the Diameter at Top 3,7 Inches, and the Solidity 70,5 solid Inches?

FIRST, the Diameter of the Bottom must be found, and then the Difference of the Solidities of the two Hoofs.

HAVING

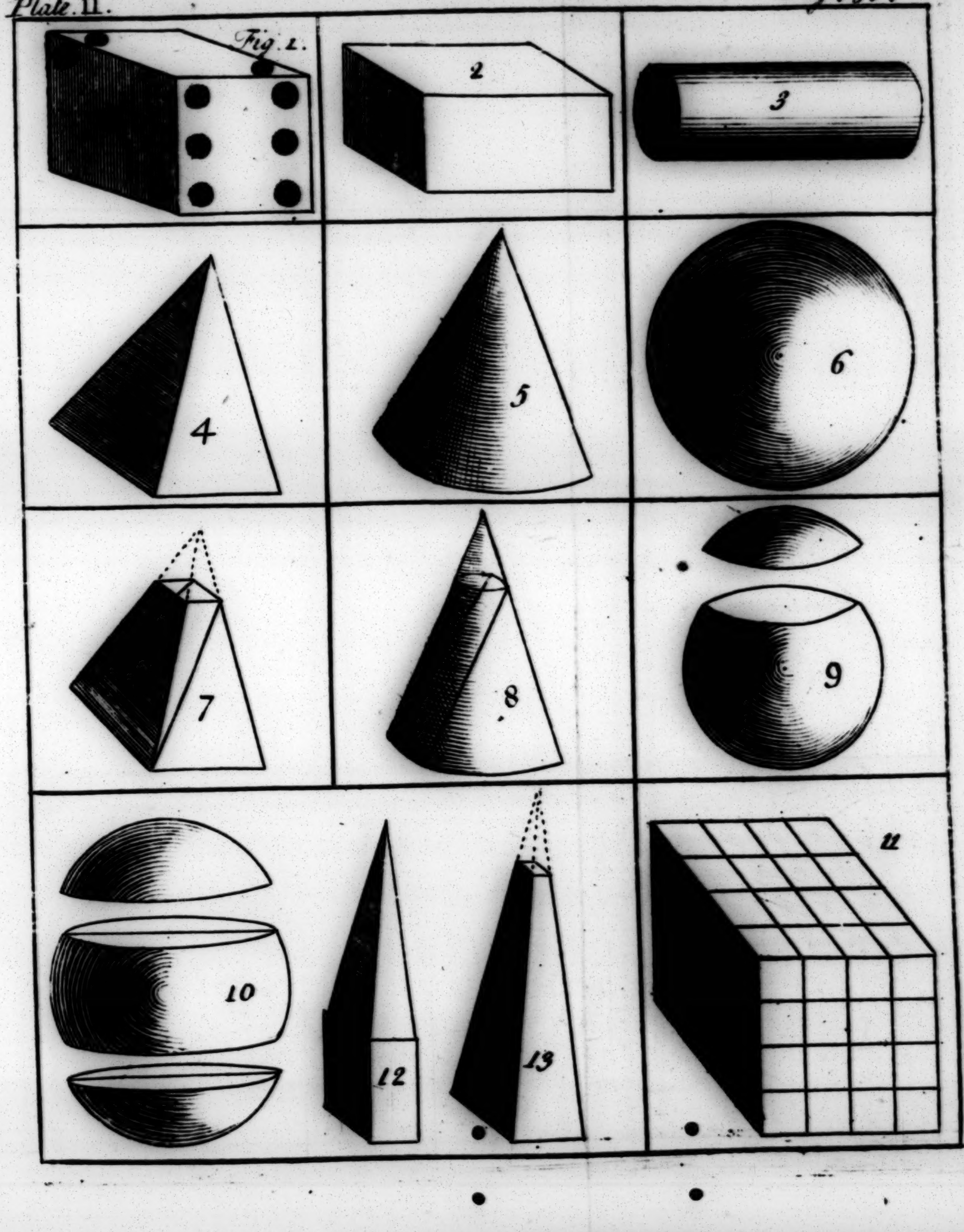
Fig. 1.
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HAVING the Solidity, Length, and one of the Diameters of any Cone's Fruſtum given, the other Diameter may be found by the following

RULE.

MULTIPLY the Length by ,2618, and divide the Solidity by this Product: From this Quotient ſubtract $\frac{3}{4}$ of the Square of the given Diameter; and extract the ſquare Root out of the Remainder, from which Root if half the given Diameter be ſubtracted, the Remainder will be the Diameter ſought.

$,2618 \times 5,7 = 1,49226$, and $\frac{70,5}{1,49226} = 47,243$, &c. and $3,7 \times 3,7 \times 3 = 41,07$, and $\frac{41,07}{4} = 10,2675 = \frac{3}{4}$ of the Square of the given Diameter.

AND $47,243 - 10,2675 = 36,9755$, whoſe ſquare Root is 6,080, &c.

AND, $\frac{3,7}{2} = 1,85$. Then $6,08 - 1,85 = 4,23$, the Diameter of the Bottom of the Pot.

AGAIN, For the lesser Hoof, or the Quantity the first Man drank.

$3,7 \times 3,7 = 13,69$, and $4,23 \times 3,7 = 15,651$, and $\frac{15,651}{2} = 7,8255$. And $13,69 + 7,8255 = 21,5155$. And $4,23 - 3,7 = ,53$, and $21,5155 - ,53 = 20,9855$. And $20,9855 \times 5,7 \times ,2618 = 31,2158$, &c. Inches, what the first Man drank.

AND $70,5 - 31,2158 = 39,2842$ Inches, what the last Man drank.

THEN $39,2842 - 31,2158 = 8,0684$ cubic Inches, the Difference of their Shares.

End of the Second Part.

APPEN-

APPENDIX.

HERE I shall shew the Measurement of some Curves, called *the Conic Sections*, and also of the Solids, generated by their Rotation on their Axes.

SECTION I.

DEFINITIONS.

I. IF a Cone be cut into two Pieces, in such a Manner, that the cutting Instrument (which is supposed flat) do always keep parallel to the Base; it is plain that the Surface of each Piece is a Circle.

II. BUT

II. BUT if the cutting Inſtrument paſſes thro' the Cone, ſo as always to keep parallel to the Side oppoſite that Side on which the Inſtrument entered ; the Surface of each Piece at this Section is called a *Parabola*.

III. IF the Cone be ſo cut, that the Inſtrument paſſes evenly through any where between the Sections of the Circle and Parabola, the Surface of each Piece at this Section is called an *Ellipſis*.

IV. IF the Cone be ſo cut, that the Inſtrument paſſes evenly any where through between the Section of the Parabola, and that Side of the Cone on which the Inſtrument entered, the Surface of each Piece at this Section is called a *Hyperbola*.

THUS, ſuppoſe ABC (*Fig.* 1. *Plate* 3.) a Cone, and D the Point on which the Inſtrument is to enter.

Firſt. THE Section DE is a *Circle*.

Secondly. THE Section DF is an *Ellipſis*.

Thirdly. THE Section DG is a *Parabola*.

Fourthly. THE Section DH is a *Hyperbola*.

THE Forms of the Curves of the *Ellipſis*, *Parabola*, and *Hyperbola*, are repreſented in the Figures 2, 3, and 4.

V. THAT Point of the Curve in any of the conic Sections, where the Curve is moſt acute, [or ſharpeſt] is called the *Vertex*.

VI. A LINE drawn through the Vertex, ſo as to divide the Area of the Section into two equal Parts, is called the *Axis of the Curve*.

Note. IN the Ellipſis this Line is called the *Tranſverſe Diameter*, or *Axis*.

VII. A LINE drawn at right Angles to the Axis, and terminated at each End by the Curve is called an *Ordinate.*

Note, THAT an Ordinate drawn through the middle of the Tranſverſe Axis in the Ellipſis, is called the *Conjugate Axis*, or *Diameter.*

VIII. IF an Ellipſis be moved round on either of it's Diameters [ſuppoſing it fixed] till the Motion end where it began; a Solid that would fill the Space thus generated, is called a *Spheroid.*

Note. IF the Ellipſis be turned on the Tranſverſe Axis, the Solid generated is called a *Prolate Spheroid*, and is nearly of the Figure of an Egg.

BUT if it be turned on the Conjugate Axis, the Solid generated is called an *Oblate Spheroid*, and is nearly of the Figure of a Bowling-Green Bowl.

IX. SUPPOSE the Axis of a Parabola or an Hyperbola be fixed, and then the Curve be turned round it's Axis, till the Motion end where it began; a Solid that would fill the Space thus generated is called a *Conoid.*

IN the Parabola it is called a *Parabolic Conoid*, or *Paraboloid.*

IN the Hyperbola it is called a *Hyperbolic Conoid*, or *Hyperboloid.*

X. IF a Parabola be turned round on an Ordinate [fixed] till the Motion end where it began; a Solid that would fill the Space thus generated, is called a *Parabolic Spindle.*

SECTION II.

Of the Ellipsis *and it's Parts.*

PROPOSITION I.

Having the Transverse and Conjugate Diameter of an Ellipsis given, to find the Area.

RULE.

MULTIPLY the Rectangle contained by the Transverse and Conjugate Diameters by ,7854, and the Product is the Area.

For the Area of every Ellipsis is a mean Proportional between the Area of two Circles, one whose Diameter is the Transverse Axis, and the other the Conjugate Axis. *Fig.* 4.

EXAMPLE I.

What is the Area of an Ellipsis, whose Transverse Axis is 24, and the Conjugate 18?

$24 \times 18 \times ,7854 = 339,2928$, the Area sought.

Hence

HENCE it is eaſy to conceive that if the Area of any Ellipſis be divided by ,7854 and that Quotient by one of the Diameters, the Reſult will be the other Diameter.

THE Segment of any Ellipſis is obtain'd thus. *Fig.* 5.

IF the Ordinate cutting off the Segment be parallel to the Tranſverſe Axis, deſcribe a Circle on the Conjugate Axis, and then it will be:

As the Area of this Circle, is to the Area of the Ellipſis, ſo is the Area of any Segment in this Circle, to the Area of it's correſponding elliptical Segment.

EXAMPLE.

THERE is an Ellipſis whoſe Tranſverſe Diameter is 12 Inches, and the Conjugate 8 Inches, I deſire to know the Area of a Segment of this Ellipſis cut 2 Inches diſtant from the Center, and parallel to the Tranſverſe Axis:

NOW it is evident, if a Circle be deſcribed on the Conjugate Axis, the Semi-chord in the Circle will be leſs than the Semi-ordinate in the Ellipſis; in order to find the Area of the Segment of an Ellipſis, there muſt be known the Ellipſis's Area, the Circle's Area, and the Area of the Circle's Segment correſponding to that of the Ellipſis.

$12 \times 8 \times ,7854 = 75,3984$, the Area of the Ellipsis.

$8 \times 8 \times ,7854 = 50,2656$, the Area of a Circle described on the conjugate Axis.

$\frac{8}{2} = 4$, the Radius. And $4 - 2 = 2$, the versed Sine. And $4 + 2 = 6$.

THEN $6 \times 2 = 12$, whose square Root is 3,46, &c. the half Chord, *per* PROPOSIT. XIV. PART. I.

$4 \times 8 \times 2 = 64$, and $14 \times 4 \times 4 = 224$, and $224 - 64 = 160$, and $2 \times 2 \times 6 = 24$, and $160 - 24 = 136$, and $4 \times 9 = 36$, and $2 \times 6 = 12$, and $36 + 12 = 48$, and $\frac{136}{48} = 2,8z$.

THEN $2,8z \times 3,46 = 9,80z$, the Area of the Segment of the Circle, *per* PROPOSIT. XVII. PART. I.

$50,2656 : 75,3984 :: 9,80z : 14,7$, the Area of the Elliptical Segment.

AFTER the same Manner would the Area be found, was the Ordinate to be given parallel to the Conjugate; only in this Case you must work with the Circle described on the Transverse Axis. *Fig*. 6.

PRO-

PROPOSITION III.

To find the Solidity of a Spheroid.
(FIG. 7.)

RULE.

FIND the Solidity of a Cylinder, whose Axis is equal to that of the Spheroid, and the Diameter of whose Base is equal to the greatest Ordinate in the Spheroid, and $\frac{2}{3}$ of that Cylinder's Solidity, is the Spheroid's Solidity.

OR thus, Multiply the Square of the greatest Ordinate by the Axis; and this Product by ,5236 gives the Spheroid's Solidity.

EXAMPLE.

WHAT is the Solidity of a Spheroid, whose Axis is 24 Inches, and the greatest Ordinate 18 Inches?

$18 \times 18 \times ,7854 \times 24 = 6107,2704$, the Solidity of the Cylinder.

AND, $6107,2704 \times 2 = 12214,5408$, and $\frac{12214,5408}{3} = 4071,5136$, the Spheroid's Solidity.

OR thus, $18 \times 18 \times 24 \times ,5236 = 4071,5136$, the Solidity as before.

S BECAUSE

BECAUSE a Spheroid is $\frac{2}{3}$ of the circumscribing Cylinder, and a Cone is $\frac{1}{3}$ of a Cylinder of the same Base and Altitude, it follows that a Spheroid is double to a Cone, whose Axis is equal to the Spheroid's Axis, and the Diameter of whose Base is equal to the Spheroid's greatest Ordinate.

PROPOSITION III.

To find the Solidity of any Frustum of a Spheroid. [Fig. 8.]

RULE.

SAY as the Solidity of the inscribed or circumscribing Sphere : is to the Solidity of the Spheroid : : so is any Segment of a Sphere : to the corresponding Part of the Spheroid.

THIS Rule will either find the Solidity of the middle Frustum or Zone, or the Solidity of a Frustum, cut from one End of one Axis, and parallel to the other Axis.

EXAMPLE I.

SUPPSOE a Spheroid whose transverse Axis was 20 Inches, and the Conjugate 16 Inches, was

was to have a Slice taken off, parallel to the Conjugate, at 6 Inches diſtant from the Center; what is the Solidity of that Piece?

$16 \times 16 \times 20 \times ,5236 = 2680,832$, the Solidity of the Spheroid.

$20 \times 20 \times 20 \times ,5236 = 4188,8$, the Solidity of the Sphere.

$\frac{20}{2} = 10$, and $10 - 6 = 4$, the verſed Sine.

AND $20 \times 3 = 60$, and $4 \times 2 = 8$, and $60 - 8 = 52$, and $4 \times 4 = 16$.

THEN $52 \times 16 \times ,5236 = 435,6352$, the Solidity of a Segment of the circumſcribing Sphere, correſponding to the Solidity of the required Spheroid's Segment.

THEN, $4188,8 : 2680,832 :: 435,6352 : 278,80$, &c, the Spheroid's Segment.

BUT if the Solidity of the Zone was required, beſides the foregoing Rule there is another much more expeditious as follows. *Fig.* 9.

RULE.

To twice the Square of the conjugate Diameter, add the Square of the Diameter of the Zone's End; this Sum multiply by the Diſtance between the Conjugate and End, and this laſt Product by $\frac{,5236}{2}$ (,2618) and the Product is the Solidity of that Part of the Zone.

If the Diameters of the Zone's two Ends are equal, double this laſt Product, gives the Solidity of the whole Zone. But if they are unequal, proceed as before, with the Zone's other End, and the two Solidities added, give the whole Solidity.

This Rule is of Uſe in finding the Solidity or Contents of a Cask, ſuppoſed to be the middle Fruſtum of a Spheroid. *Fig.* 9.

EXAMPLE II.

Suppose a Cask (the middle Fruſtum of a Spheroid) the Diameter at the Bung (or Conjugate) is 16, the Diameter of either of the Heads (or Ends) is 12,8, and the Length is 12; what is the Content in ſolid Meaſure?

This Example which is the middle Fruſtum of the Spheroid in the foregoing Example, (whoſe Tranſverſe is 20) I ſhall operate by both the Rules.

Firſt, By the firſt Rule.

$20 \times 20 \times 20 \times ,5236 = 4188,8$, the Sphere's Solidity.

$16 \times 16 \times 20 \times ,5236 = 2680,832$, the Solidity of the Spheroid, as before.

$20 \times 20 \times ,7854 \times 2 = 628,32$, and $16 \times 16 \times ,7854 = 201,0624$, and $628,32 + 201,0624 = 829,3824$, and $829,3824 \times \frac{12}{3} = 3317,5296$, the Solidity of the circumſcribing Sphere's Zone, *per* Theor. XIII. Part II.

Then 4188,8 *Spher.S.* : 2680,832 *Spheroid's S.* : : 3317,5296 *Spher.Z.S.* : 2123,218944, the Solidity of the Cask, or Spheroid's Zone.

Secondly,

Secondly, By the ſecond Rule.

$16 \times 16 \times 2 = 512$, the double Square of the Conjugate, or Bung Diameter.

$12,8 \times 12,8 = 163,84$, and $512 + 163,84 = 675,84$, and $675,84 \times \frac{12}{2} = 4055,04$, and $4055,04 \times ,2618 \times 2 = 2123,218944$, the Cask's Solidity as before.

EXAMPLE III.

WHAT is the Solidity of a Cask, taken as the middle Zone of a Spheroid; ſuppoſing the Bung Diameter 3 Feet, the Head 2 Feet, and the Length 5 Feet?

$3 \times 3 \times 2 = 18$, and $2 \times 2 = 4$, and $18 + 4 = 22$, and $22 \times 5 \times ,2618 = 28,798$ Feet, the Solidity ſought after.

Note, THE foregoing Rule is calculated for finding the Solidity of but one Side of the Zone, ſuppoſing it cut through at the Conjugate; but if the two Ends are equal, inſtead of multiplying by the Diſtance between the Conjugate and End, multiply by the Zone's whole Length, *&c.* (*i. e.* Breadth) and the Product will be the whole Solidity.

SECTION III.

Of a Parabola *and it's Parts.*

PROPOSITION IV.

To find the Area of a Parabola, by having the Ordinate and Axis given. [Fig. 10]

RULE.

MULTIPLY the Ordinate by the Axis, and this Product by 2, and the last Product divide by 3, and the Quote is the Area.

For every Parabola is $\frac{2}{3}$ of it's circumscribing Parallelogram.

EXAMPLE.

What is the Area of a Parabola, whose Axis is 12, and the Ordinate 16?

$16 \times 12 \times 2 = 384$, and $\frac{384}{3} = 128$, the Area sought after.

PRO-

PROPOSITION V.

To find the Solidity of a Parabolic Conoid, by having the Diameter of the Base and the Altitude given. [Fig. 11]

RULE.

MULTIPLY the Area of the Base by half the Height, and the Product is the Solidity; for every parabolic Conoid is half of it's circumscribing Cylinder.

EXAMPLE.

WHAT is the Solidity of a Paraboloid, the Diameter of whose Base is 20, and the Height 12?

$20 \times 20 \times ,7854 \times \frac{12}{2} = 1884,76$, the Solidity sought after.

PROPOSITION VI.

To find the Solidity of the lower Frustum of a Paraboloid. [Fig. 12]

RULE.

MULTIPLY the Sum of the Squares of the Diameters of the two Ends, by ,3927 ($= \frac{1}{8}$ of 3,1416) and multiply the Product by the Length, gives the Solidity.

EXAM-

EXAMPLE.

A FRUSTUM of a Paraboloid has one Diameter at the End 40, and the other End Diameter is 24, and the Length between the Diameters, or Height, is 12; what is the Solidity?

$40 \times 40 = 1600$, and $24 \times 24 = 576$, and $1600 + 576 = 2176$, and $2176 \times ,3927 \times 12 = 10254,1824$, the Solidity sought after.

PROPOSITION VII.

To find the Solidity of a Parabolic Spindle, by having the Length and the Diameter of the greatest circumscribing Circle given. [Fig. 13]

RULE.

MULTIPLY the Square of the Diameter by ,418 (*i. e.* $\frac{2 \times 3,1416}{15}$) and this Product multiply by the Length, gives the Solidity.

EXAMPLE.

EXAMPLE.

THERE is a parabolic Spindle, whose Length is 9 Feet, and the Diameter of the greatest Circle is 40 Inches; what is the Solidity?

40 *In.* = 3,3 *F.* Then 3,3 × 3,3 × ,418 × 9 = 418,7 Feet, the Solidity.

Note, EVERY Parabolic Spindle is $\frac{8}{15}$ of it's circumscribing Cylinder.

PROPOSITION VIII.

To find the Solidity of the middle Frustum, or Zone of a Parabolic Spindle. [Fig. 14]

RULE.

To twice the Square of the greater Diameter add the Square of the lesser Diameter, and this Sum multiply by ,7854; from this Product subtract $\frac{2}{5}$ of the Area of a Circle whose Diameter is equal to the Difference between the greater and lesser End's Diameters; and this Remainder multiply by $\frac{1}{3}$ of the Length gives the Solidity.

THIS

This Propoſition is of Uſe in finding the Contents of ſuch Casks whoſe Figure is that of the middle Zone of a Parabolic Spindle.

EXAMPLE.

There is a Cask whoſe Figure is the middle Zone of a parabolic Spindle ; the Bung Diameter 44 Inches, the Head Diameter 28 Inches, and the Length 5 Feet ; what is the Solidity or Contents thereof ?

44 *In.* $= 3,\cancel{6}$ *F.* and 28 *In.* $= 2,\cancel{3}$ *F.* and $3,\cancel{6} - 2,\cancel{3} = 1,\cancel{3}$ *F.* the Difference between the Diameters.

Then $3,\cancel{6} \times 3,\cancel{6} = 13,\cancel{4}$, and $13,\cancel{4} \times 2 = 26,\cancel{8}$ = twice the Square of the Bung Diameter.

$2,\cancel{3} \times 2,\cancel{3} = 5,\cancel{4}$, the Square of the Head Diameter.

And $26,\cancel{8} + 5,\cancel{4} = 32,\cancel{3}$, and $32,\cancel{3} \times ,7854 = 25,3946$, and $1,\cancel{3} \times 1,\cancel{3} \times ,7854 = 1,3962\cancel{6}$ and $\frac{1,3962\cancel{6} \times 2}{5} = ,55850\cancel{6}$, and $25,3946 - ,55850\cancel{6} = 24,8609\cancel{3}$, and $24,8609\cancel{3} \times \frac{5}{3}$ $(1,\cancel{6}) = 41,4101\cancel{5}$ Feet, the Solidity.

SECT.

SECTION IV.

Of the HYPERBOLA.

THERE may ſeveral Hyperbolical Curves be drawn through the ſame Vertex, and all paſs through the Extremities of the ſame Ordinate, kept always at the ſame Diſtance from the Vertex.

HENCE it will be very eaſy to conceive, that no ſettled Rules can be given for finding the Area of an Hyperbola [like thoſe for the Ellipſis and Parabola] without taking into Conſideration ſome other Lines beſide the Axis and Ordinate, which I ſhall omit, this Treatiſe not being intended to inſtruct the Reader in the Properties of the Conic Sections.

BUT here note, that a certain ingenious Author hath aſſerted, that every Appolonian (or Conical) Hyperbola is greater than half, and leſs than three fourths of it's circumſcribing Parallelogram: If ſo, it will be accurate enough to take all Hyperbola's at a mean; thus, ſay every Hyperbola is $\frac{5}{8}$ of it's circumſcribing Parallelogram; therefore multiply the Product of the Axis and Ordinate by 5, and this Product divide by 8, will give the Area nearly.

EXAM-

EXAMPLE.

WHAT is the Area of a Hyperbola, whoſe Axis is 28, and the Length of the Ordinate drawn that Diſtance from the Vertex is 44?

$\overline{44 \times 28} \times 5 = 6175$, then $\frac{6175}{8} = 771{,}875$, the Area.

As there can be no Rules given (whoſe Praxis is eaſy) for finding the Area of an Hyperbola, ſo there can be no eaſy Rules given for finding the Solidities of the Solids, that may be generated by the Rotation of the Hyperbola round it's Axis or Ordinates.

SECTION V.

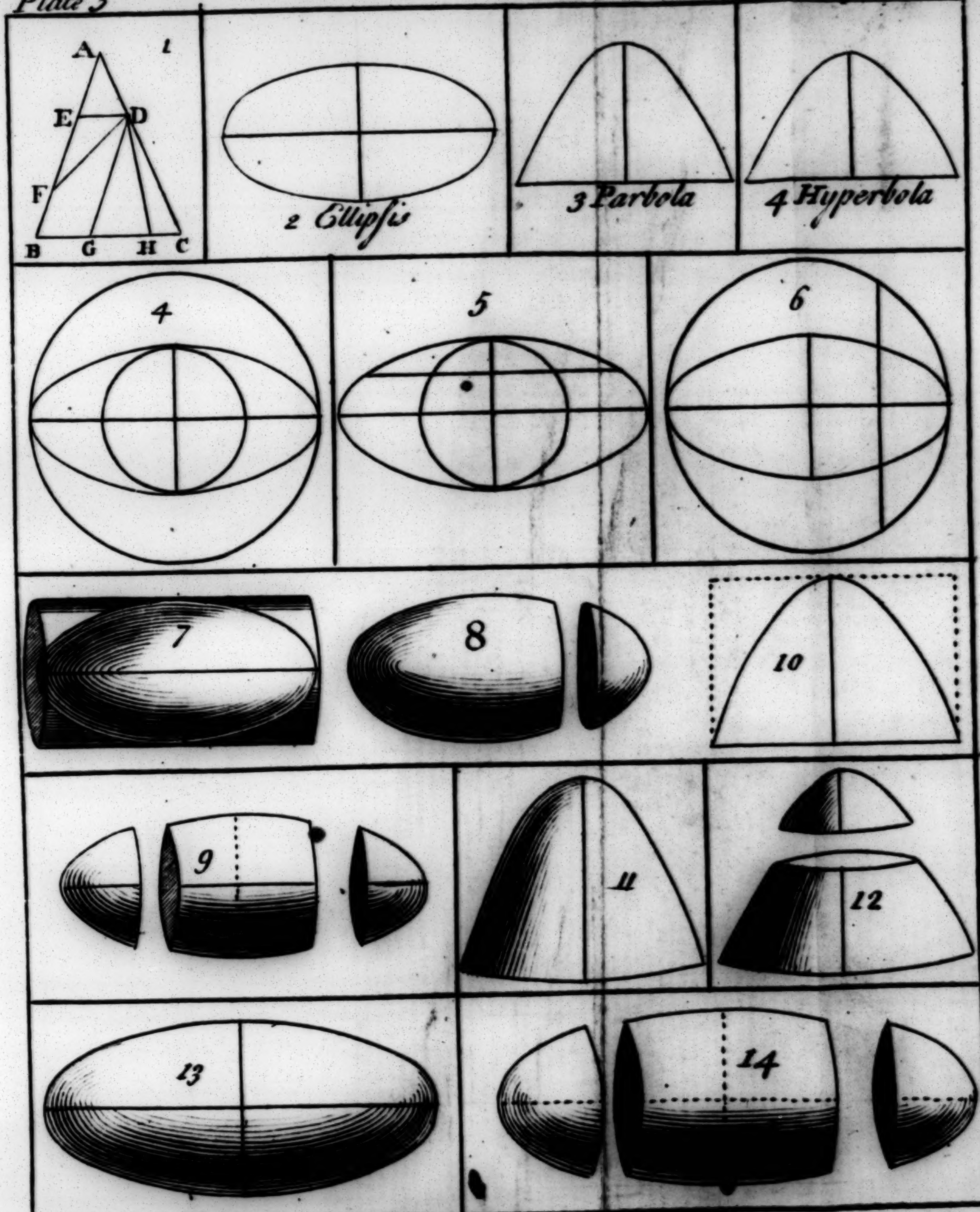
Of REGULAR SOLIDS.

THOSE Solids whoſe Sides being all equal, and the ſeveral Superficies bounding the Solid all alike and equal, and which would be ſo contain'd within a Sphere, that it's Angles would all touch the Superficies of the Sphere, are called *Regular Solids*.

THERE are five regular Solids.

I. A TETRAEDRON, which is a Solid contain'd under four equal and equilateral Triangles. ——Conſequently ſuch a Solid is a Pyramid, ſtanding on an equilateral triangular Baſe, the Superficies whereof will be equal to four times the Area of the Baſe.

II. AN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2 Ellipsis
3 Parbola
4 Hyperbola
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II. An Octaedron; which is a Solid, composed of 8 equal Pyramids, whose Vertices all meet in a Point, (which Point is the Center of the Solid) the Base of each being an equilateral Triangle, and equal one to the other——Therefore the Superficies is equal to eight times the Area of one Triangle, and the Solidity equal to the Solidities of the eight composing Pyramids.

III. A Hexaedron or Cube; which has been already defined.

IV. A Dodecaedron; which is a Solid composed of twelve equal Pyramids, whose Vertices all meet in a Point (which Point is the Center of the Solid) the Base being an equilateral Pentagon, and equal to one another.——Therefore the Superficies is equal to twelve times the Area of one Pentagon; and the Solidity equal to the Solidities of the twelve composing Pyramids.

V. An Icosaedron; which is a Solid composed of 20 equal Pyramids, whose Vertices meet all in a Point, (which Point is the Center of the Solid) the Base of each Pyramid being an equilateral Triangle, and equal to one another.——Therefore the Superficies is equal to twenty times the Area of one Triangle; and the Solidity equal to the Solidities of the twenty composing Pyramids.

But because it is somewhat difficult to find the perpendicular Heights of the composing Triangles in each Solid, therefore the following Table is so framed that the Solidity and Superficies of any of the regular Solids may be easily attained, by having the Side given.

A TABLE shewing the Superficies and Solidity of any of the regular Solids, whose Side is 1.

Bodies Names.	Solidity.	Superficies.
TETRAEDRON.	0,117851	1,732051
OCTAEDRON.	0,471404	3,464102
HEXAEDRON.	1,000000	6,000000
ICOSAEDRON.	2,181695	8,660254
DODECAEDRON.	7,663119	20,645729

RULE I.

MULTIPLY the Square of the given Side, by the Tabular Number under Superficies against the given Name, and the Product is the Superficies.

RULE II.

MULTIPLY the Cube of the given Side by the Tabular Number under Solidities against the given Name, and the Product is the Solidity.

EXAMPLE.

WHAT is the Solidity and Superficies of an Icosaëdron, one of whose Sides is 40 Inches?

LOOK in the Table under Solidity against the Name *Icosaëdron*, you will find, 2,181695. Then,

$40 \times 40 \times 40 \times 2,181695 = 139628,48 =$ the Solidity of an Icósaëdron, whose Side is 40.

AGAIN

AGAIN, Look in the Table under Superficies, and againſt the Name *Icoſaëdron* you will find, 8,660254. Then $\overline{40 \times 40} \times 8,660254 =$ 13856,4064, the Superficies; proceed in the ſame Manner for the other Solids.

Now as all theſe regular Solids may be inſcribed within a Sphere; if your Curioſity ſhould lead you at any Time to know what would be the Sides, Solidities, and Superficies of any of the forementionʼd Bodies inſcribed in any given Sphere, you may eaſily ſatisfy yourſelf from the following Table and the foregoing one.

A TABLE ſhewing the Proportions of the Sphere, and the five regular Figures inſcribed in the ſame.

Bodies Names	Sides.	Superficies.	Solidities.
TETRAEDRON.	1,62299	4,6188	0,15132
HEXAEDRON.	1,1547	8,0000	1,5396
OCTAEDRON.	1,41421	6,9282	1,3333
DODECAEDRON.	0,71364	10,51462	2,78516
ICOSAEDRON.	1,05146	9,57454	2,53615

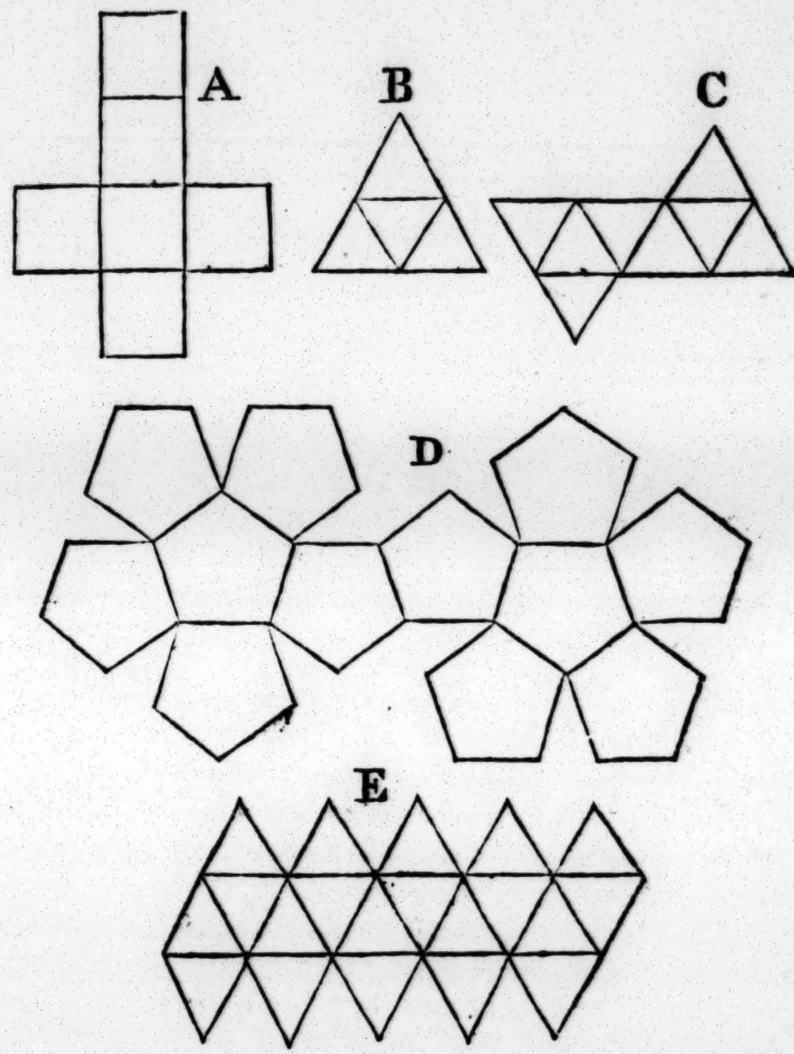
The Sphere's Diameter muſt be 2, *to circumſcribe theſe Solids, and ſuch a Sphere's Superficies will be* 12,56637, *and the Solidity* 4,1879.

EXAMPLE.

WHAT will be the Sides, Superficies, and Solidities of the regular Solids inscribed in a Sphere of 12 Inches in Diameter?

2 : 1,62299 : : 12 : 9,73794 = Tetraëdron's Side.
2 : 1,1547 : : 12 : 6,9282 = Hexaëdron's Side.
2 : 1,41421 : : 12 : 8,48526 = Octaëdron's Side.
2 : 0,71364 : : 12 : 4,28184 = Dodecaëdron's Side.
2 : 1,05146 : : 12 : 6,30876 = Icosaëdron's Side.

THUS having got the Length of the Sides, the Solidities and Superficies may easily be found either by this Table or the foregoing one.



If five ſuch Figures as theſe be cut in Paſt-board, or any other pliable Matter, and if the Lines be cut half through, and the Sides turned up and glued together, they will repreſent the regular Solids: *viz.* A, the Cube or Hexaëdron; B, the Tetraëdron; C, the Octaëdron; D, the Dodecaëdron; and E, the Icoſaëdron.

SECTION VI.

Of GROIN ARCHES.

WHEN two Arches in a Building interſect each other at right Angles, ſuch Interſections make what are called *Groin Arches.*

SUCH are the Arches under the Treaſury in the Park, *Covent Garden*, and in many other Places; and how to find the Solidity and Superficies of any ſuch Arches, ſuppoſing them the Arches of Circles, obſerve the following

RULES.

I. MULTIPLY the Area of the Baſe whereon ſuch an Arch ſtands, by the Height, and two thirds of this Product is the Solidity.

II. AND twice the Area of the Baſe of ſuch an Arch, is the Superficies.

Note, THESE Rules ſuppoſe the interſecting Arches equal, and it's Baſe to be a Square.

BUT if the Arches are not equal, or if they be Segments of Ellipſes, the Rules for finding the Solidity and Superficies of ſuch Groins, are difficult, and are not eaſily communicated to a Perſon not more than an Arithmetician, and therefore I ſhall not attempt it.

EXAM-

EXAMPLE I.

WHAT is the Solidity and Superficies of a semicircular Groin Arch, whose Diameter is 12 Feet?

$$12 \times 12 \times \frac{12}{2} = 864, \text{ and } 864 \times 2 = 1728,$$

and $\frac{1728}{3} = 567$, the Solidity of the Groin.

AND, $12 \times 12 \times 2 = 288$, the Superficies.

BUT if it were Brick or Stone Work, and you wanted to find the Solidity thereof; then

MULTIPLY the Area of the Base by the Height [taking in the Thickness of the Work at Top] and from this Product subtract the Solidity of the Groin (found as before) and it leaves the Solidity of the Work.

EXAMPLE II.

SUPPOSE the Roof of a Building was supported by the Side-Walls and two Rows of Columns, 11 in each Row, the two Extreams of each Row were sunk in the End-Walls, and the middle Roof turned off the Colums in the Arch of a Circle, which Roof was intersected by 10 Arches of Circles on each Side, coming from as many Windows in the Sides of the Building, making thereby as many Groin Arches; the Top of the Columns from whence the Arch turned off was square, each Side whereof was 2 Feet; the Chord of each Arch was 20 Feet, and the versed Sine (or Height) 8 Feet: Now supposing the Top of the Roof made flat, by filling up the Spandils, (or Interstices of the Arches without) and the Work continued till it was raised 2 Feet above the Top of the Groin, and

supposing

supposing the Columns were distant from the Sides of the Building 12 Feet; what would be the Solidity and Superficies of this Roof?

First, For the Groins, $20 \times 20 \times 8 = 3200$, and, $\frac{3200 \times 2}{3} = 2133,\dot{3}$, the Solidity of one Groin.

And, $2133,\dot{3} \times 10 = 21333,\dot{3}$, the Solidity of all the Groins.

Again, $20 \times 20 \times \overline{8+2} = 4000$, the Solidity of one Parallelopipedon, including one Groin; and $4000 \times 10 = 40000$, the Soliddity of all the Parallelopipedons including all the Groins.

Then, $40000 - 21333,\dot{3} = 18666,\dot{6}$, the Solidity of all the Work above the Groins.

Now each Column being 12 Feet Distance from the Wall, and the Side of the Square on the Top of each Column 2 Feet; therefore the Arch from the Windows to each Groin will be Segments of hollow Cylinders cut parallel to it's Axis, it's Length being 14 Feet; and adding the Lengths of two such together (they being opposite) make 28 Feet: Then finding the Area of the Segment of a Circle, whose Chord is 20 Feet, and versed Sine 8, *viz. per* Propos. XVII. Part I. *Meas.*

$\frac{20}{2} = 10$, and $10 \times 10 = 100$, and $\frac{100}{8} = 12,5$

and $12,5 + 8 = 20,5$, and $\frac{20,5}{2} = 10,25 =$ to the Radius.

And, $10,25 - 8 = 2,25$, the Difference.

Then, $10,25 \times 10,25 \times 14 = 1470,875$, and $10,25 \times 8 \times 2,25 = 184,5$, and $1470,875 - 184,5 = 1286,375$, and $2,25 \times 2,25 \times 6 = 30,375$, and $1286,375 - 30,375 = 1256$, and $10,25 \times 9 = 92,25$, and $2,25 \times 6 = 13,5$, and $9225 + 13,5 = 105,75$,

$= 105{,}75$, and $\frac{1256}{105{,}75} = 11{,}877$, and $11{,}877 \times \frac{20}{2} = 118{,}77$, the Area of the Segment.

THIS Area, *viz.* $118{,}77 \times 28$, the Length of two oppoſite cylindrical Segments, gives 3325,56, the Solidity of one ſuch Segment: And $3325{,}56 \times 10 = 33255{,}6$, the Solidity of all the Segments.

AND, $20 \times \overline{8 + 2} \times 28 = 5600$, the Solidity of one rectangular Priſm, including one Cylinder's Segment.

AND, $5600 \times 10 = 56000$ = Solidity of all ſuch Priſms.

AND, $56000 - 33255{,}6 = 22744{,}4$, the Solidity of all the Work over the Cylindrical Segments.

AGAIN, $12 + 2 + 20 + 2 + 12 = 48$, the Breadth of the Building.

AND, $48 \times 10 \times 2 = 960$, the Solidity of the Wall ſeparating two Groins, out of which take $118{,}77 \times 2 = 237{,}54$, the Solidity of the Segment between two Groins, leaves 722,46, the Solidity of one parting Wall; and $722{,}46 \times 10 = 7224{,}6$, the Solidity of all.

THEN, $18666{,}\cancel{6} + 22744{,}\cancel{4} + 7224{,}6 = 48635{,}\cancel{6}$, the Solidity of the whole Roof.

AGAIN, $20 \times 20 \times 2 \times 10 = 8000$, the Superficies of all the Groins.

AND, $\frac{20}{2} = 10$, and $10 \times 10 = 100$, and $8 \times 8 = 64$, and $100 + 64 = 164$, whoſe ſquare Root is 12,8, *per* PROP. XV. PART. I. *Meaſ.* and $\frac{\overline{12{,}8 \times 8} - 20}{3} = 27{,}4\cancel{6}$, the Length of the Arch of one cylindric Segment.

And $27{,}4\cancel{6} \times 28 \times 10 = 7690{,}\cancel{6}$, the Superficies of all the Cylindric Segments. AND

And $12 + 12 = 24$, and $24 \times 2 \times 10 = 480$, the Superficies of the Parts between the cylindric Segments, and the Wall, and Columns.

And, $27,46 \times 2 \times 10 = 549,3$, the Superficies of the Arches ſeparating the Groins.

Then $8000 + 7690,6 + 480 + 549,3 = 16720$, the whole Superficies within the Roof.

The following Table ſhews the ſpecific Gravity or Weight of Metals and other Bodies in Troy Weight *and* Avoirdupoiſe Weight.

		Oz. Troy.		Oz. Avoirdup.
A Cubic Inch of	Fine Gold - -	10,359273	=	11,365602
	Standard Gold -	9,962625	=	10,930422
	Quickſilver - -	7,384411	=	8,101753
	Lead - - -	5,984010	=	6,553885
	Fine Silver - -	5,850035	=	6,418324
	Standard Silver -	5,556769	=	6,096596
	Roſe Copper -	4,747121	=	5,208369
	Plate Braſs - -	4,404273	=	4,832116
	Caſt Braſs - -	4,272409	=	4,630300
	Steel - - - - -	4,142127	=	4,544505
	Common Iron -	4,031361	=	4,422979
	Block-Tin - -	3,861519	=	4,236638
	Fine Marble - -	1,429411	=	1,568859
	Common Glaſs -	1,360841	=	1,493037
	Alabaſter - -	0,988456	=	1,084477
	Dry Ivory - -	0,962083	=	1,055542
	Dry Box-Wood -	0,543282	=	0,596057
	Sea Water - -	0,542742	=	0,594894
	Common clear ditto	0,527458	=	0,578697
	Red Wine - -	0,523766	=	0,574646
	Proof Spir. of Brandy	0,489268	=	0,536796
	Sound dry Oak -	0,489008	=	0,536569
	Lint-Seed Oyl -	0,491591	=	0,539345
	Olive Oyl - -	0,481569	=	0,528350

From

From this Table it will be very easy to find the Weight of any Body whose Name is mention'd therein, by having the Solidity in cubic Inches given. For multiply the cubic Inches contain'd in any Body (whose Name is in the Table) by the tabular Number, and the Product will be the Weight, either in Troy or Averdupoise, as you take the Number.

EXAMPLE I.

There is a Piece of Oak, of a rectangular Form, the Length is 56 Inches, Breadth 18, and Depth 12 Inches; what is the Weight thereof in Averdupoise Weight?

$18 \times 12 \times 56 = 12096$, the solid Inches; and 1 Inch weighs 0,536569 Ounces Averdupoise: Therefore, $12096 \times ,536569 = 6490,338624$ *oz.* which is 3 *C.* 2 *qrs.* 13 lb 10 *oz* Averdupoise.

Hence it is very easy to find the Solidity by having the Weight given; for divide any given Weight in Ounces by it's respective tabular Number, and it gives the Solidity in Inches.

If you would know the Weight of any Body, when immersed or put into Water, find the Weight of a Quantity of Water equal to the Quantity (or Solidity) of the Body given, and the Weight of Water subtracted from the Weight of the given Body will be it's Weight when immersed in Water.

EXAM-

EXAMPLE II.

WHAT Weight will sustain a cubic Foot of Iron when put into Water?

4,422979 × 1728 = 7642,907712, the Weight of a cubic Foot of Iron.

,578697 × 1728 = 999,988416, the Weight of a cubic Foot of Water.

THEN, 7642,907712 — 999,988416 = 6642,919296 Ounces; and such a Weight of any Matter will sustain a cubic Foot of Iron, when hanging in Water.

THE Solidity of any Body however irregular may be obtained by putting the Body in some Vessel, whose Sides are parallel, and then just cover it with Water, and find the Solidity of that Space the Water occupies, with the Body in it; after which take the Body out, letting the Water drain off: And observing exactly how much the Water has fell; then finding the Solidity of the Space occupied by the Water alone, and the Difference between this and the former Space will be the Solidity of the Body that was immersed.

SECTION VII.

Of the CONSTRUCTION *and* USE *of* DECIMAL TABLES.

DECIMAL TABLES ready calculated save much Trouble, and there being frequent Use thereof in this Treatise, and in many more, where any

Mathematical Business is concerned that regards Calculation, I thought it would not be amiss to insert a more compleat Set than as yet I have been able to meet with.

THE Construction of these Tables is performed in the same Manner, as is shewn in the Introduction, SECT. II. *Page* 8.

CONSTRUCTION *of* TABLE I.

I. THE Shillings of a Pound Sterling, *viz.* 19, 18, 17, 16, *&c.* are separately divided by 20, and the several Quotients are the Decimals of their respective Shillings.

II. THE Decimal of 1 Shilling is divided by 12, (the Pence in a Shilling) and the Quote is the Decimal of a Penny; which Decimal of a Penny multiply'd separately by the several Pence, gives their respective Decimals.

III. THE Decimal of a Penny is divided by 4, (the Farthings in a Penny) and the Quote is the Decimal of a Farthing, which, with multiplying by 2 and 3, gives the other Decimals of Farthings.

CONSTRUCTION *of* TABLE II.

I. 1 Ounce is divided by 12, (the Number of Ounces in 1 Pound Troy) and the Quotient is the Decimal of 1 Ounce, which multiply'd by the other Ounces, gives their respective Decimals.

II. The Decimal of one Ounce is divided by 20, (the Penny Weights in one Ounce) and the Quote is the Decimal of 1 Penny Weight, which multiply'd by the other Penny Weights gives their respective Decimals. And from these are the Decimals of Grains, constructed in the same Manner.

Construction of Table III.

I. Because 20 Hundred is one Ton; therefore the Decimals of the Shillings will serve for the Hundred Weights, supposing 1 Ton the Integer.

II. The Decimal of 1 Shilling (*i. e.* 1 Hundred Weight) is divided by 4 (the Quarters in 1 Hundred Weight) and the Quote is the Decimal of 1 Quarter, from whence the Decimals of the other Quarters are obtained, as are also the Decimals of Pounds.

Hundred Weight *the* Integer.

The Decimals of the Quarters are ,25 ,5 and ,75, and dividing the Decimal of 1 Quarter by 28, (the Pounds in a Quarter) gives the Decimal of 1 Pound; from whence the Decimals of the other Pounds are obtained, as are also the Decimals of Ounces; and from them the Decimals of Drams.

ONE POUND *the* INTEGER.

1 OUNCE is divided by 16, (the Ounces in 1 Pound) and it gives the Decimal of 1 Ounce, from whence the Decimals of the other Ounces are obtained, as are alſo the Decimals of Drams.

AFTER the ſame Manner are the other Tables of Meaſures and Time conſtructed, having always a due Regard how many of a leſſer Denomination are contained in a ſuperiour one.

IN the Conſtruction of theſe Tables there are ſome little Liberties taken, which is, As the Tables are only carry'd to 6 Places, (excepting in ſome Particulars) whenever the ſeventh Figure would have been more than a 5, the ſixth Place has been increaſed with Unity, otherwiſe, the ſixth Place is given as it turns out in the Operation. And univerſally, the laſt Place is always increaſed with Unity, if the next ſucceeding Place would be more than a 5.

THE Uſe of theſe Tables are obvious, but to prevent all Doubts, I will here inſert two or three.

EXAMPLE I.

WHAT is the Decimal of 7 *oz.* 16 *dwt*, 18 *grs.*?
In TABLE II.

againſt 7 *oz.* is ,5833333
againſt 16 *dwt.* is ,0666666
againſt 18 *grs.* is ,0031250

Theſe added, their Sum is ,653125, the Decimal required.

EXAMPLE

EXAMPLE II.

WHAT is the Decimal of 46 Gallons and 5 Pints?

In TABLE IV.

Against	40 Gallons	is	,634920
Against	6 Gallons	is	,395238
Against	5 Pints	is	,009921

These added, their Sum is ,740079, the Decimal required.

AFTER the same Manner are Decimals of other enominations collected.

N.B. IF you find not the Number you want in any of the Tables, take it out at twice, thus: For 17 *dwt.* add 10 and 7 together, and the like for any other.

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TABLE I.
Coin, 1 lb the Integer.

S.	Dec.	S.	Dec.
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,1	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence.	Decimals.
11	,04583 .
10	,0416 . .
9	,0375 . .
8	,03 . . . .
7	,02916 .
6	,025 . . .
5	,02083 .
4	,016 . . .
3	,0125 . .
2	,0013 . .
1	,00416 .

Farthings.	Decimals.
3	,003125
2	,002083
1	,0010416

Note. *The following Table of Ounces will serve for Inches and any thing, where 12 is the Integer.*

TABLE II.
Troy Weight.
1 lb the Integer.

Ounces.	Decimals.
11	,916 . . .
10	,83 . . . .
9	,75 . . . .
8	,6 . . . . .
7	,583 . . .
6	,5 . . . . .
5	,416 . . .
4	,3 . . . . .
3	,25 . . . .
2	,16 . . . .
1	,083 . . .

Dwts.	Decimals.
10	,0416 . .
9	,0375 . .
8	,03 . . . .
7	,02916 .
6	,025 . . .
5	,02083 .
4	,016 . . .
3	,0125 . .
2	,0083 . .
1	,00416 .

Grs.	Decimals
10	,001736
9	,001563
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

TABLE III.

Avoirdup. 1 Ton the Integ.

Qrs. Cws.	*Decimals.*
3	,0375 . .
2	,025 . . .
1	,0125 . .

Pounds.	*Decimals.*
20	,008928
10	,004464
9	,004018
8	,003571
7	,003125
6	,002678
5	,002232
4	,001785
3	,001339
2	,000892
1	,000446

1 Pound the Integer.

Ounces.	*Decimals.*
10	,625 . . .
9	,5625 . .
8	,5 . . . . .
7	,4375 . .
6	,375 . . .
5	,3125 . .
4	,25 . . . .
3	,1875 . .
2	,125 . . .
1	,0625 . .

TABLE III.

Avoirdup. 1 ℔ the Integ.

Drams.	*Decimals.*
10	,039062
9	,035156
8	,031250
7	,027344
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

1 Cwt. the Integer.

Pounds.	*Decimals.*
20	,178571
10	,089285
9	,080357
8	,071428
7	,0625
6	,053571
5	,044642
4	,035714
3	,026785
2	,017857
1	,008928

Ounces.	*Decimals.*
10	,005580
9	,005022
8	,004464
7	,003906
6	,003348
5	,002790
4	,002232
3	,001674
2	,001116
1	,000558

TABLE III.

Avoirdupoise.

1 Cwt. the Integer.

Drams.	*Decimals.*
10	,000349
9	,000314
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139
3	,000104
2	,000069
1	,000034

TABLE IV.

Liquid Measure.

1 Tun the Integer.

Gallons.	*Decimals.*
200	,793651
100	,396825
90	,357141
80	,317460
70	,277....
60	,238095
50	,198412
40	,158730
30	,119047
20	,079365
10	,039682
9	,035714
8	,031746
7	,027...
6	,023809
5	,019841
4	,015873
3	,011904
2	,007936
1	,003968

TABLE V.

Liquid Measure.

1 Tun the Integer.

Pints.	*Decimals.*
7	,003472
6	,002976
5	,002480
4	,001984
3	,001488
2	,000992
1	,000496

1 Hogshead the Integer.

Gallons.	*Decimals.*
60	,952381
50	,793651
40	,634921
30	,476190
20	,317460
10	,158730
9	,142857
8	,126984
7	,1.....
6	,095238
5	,079365
4	,063492
3	,047619
2	,031746
1	,015873

Pints.	*Decimals.*
7	,013889
6	,011905
5	,009921
4	,007937
3	,005952
2	,003968
1	,001984

TABLE V.

Long Measure.

1 *Mile the Integer.*

Yards.	*Decimals.*
1000	,568182
900	,511364
800	,454545
700	,397727
600	,340909
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051136
80	,045454
70	,039772
60	,034090
50	,028409
40	,022727
30	,017045
20	,011363
10	,005681
9	,005113
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001136
1	,000568

TABLE V.

Long Measure.

1 *Mile the Integer.*

Feet.	*Decimals.*
2	,0003787
1	,0001894
Inches.	*Decimals.*
9	,0001421
6	,0000947
3	,0000474
1	,0000158

1 *Yard the Integer.*

Feet.	*Decimals.*
2	,6 . . . . .
1	,3 . . . . .
Inches.	*Decimals.*
11	,305 . . .
10	,27 . . . .
9	,249 . . .
8	,2 . . . . .
7	,194 . . .
6	,16 . . . .
5	,138 . . .
4	,1 . . . . .
3	,083 . . .
2	,05 . . . .
1	,027 . . .
Qrs. Inch.	*Decimals.*
3	,02083 .
2	,0138 . .
1	,00694 .

TABLE VI.

Time.

1 Year the Integer.

Days.	*Decimals.*
300	,821918
200	,547945
100	,273973
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054792
10	,027397
9	,024657
8	,021917
7	,019178
6	,016438
5	,013698
4	,010959
3	,008219
2	,005479
1	,002740

TABLE VI.

Time.

1 Year the Integer.

Hours.	*Decimals.*
20	,002282
11	,001141
9	,001027
8	,000913
7	,000799
6	,000685
5	,000571
4	,000456
3	,000342
2	,000228
1	,000114
Minutes.	***Decimals.***
50	,000951
40	,0000761
30	,00005706
20	,00003804
10	,00001902
9	,00001711
8	,00001521
7	,00001331
6	,00001141
5	,00000951
4	,00000761
3	,00000571
2	,00000380
1	,00000190
Qrs. Min.	***Decimals.***
3	,00000142
2	,00000095
1	,00000047

TABLE VI.

Time.

1 Day the Integer.

Hours.	*Decimals.*
20	,83 . . . .
10	,416 . . .
9	,375 . . .
8	,3 . . . . .
7	,2916 . .
6	,25 . . . .
5	,2083 . .
4	,16 . . . .
3	,125 . . .
2	,083 . . .
1	,0416 . .

Minutes.	*Decimals.*
50	,03472
40	,027 . . .
30	,02083 .
20	,0138 . .
10	,00694 .
9	,00625 .
8	,005 . .
7	,004861
6	,00416 .
5	,003472
4	,0027 . .
3	,002083
2	,00138 .
1	,000694

Seconds.	*Decimals.*
45	,0005208
30	.0003472
15	,0001736

TABLE VII.

Various Measures.

1st. *Cloth Measures.*

1 Yard the Integer.

Qrs.Yard.	*Decimals.*
3	,75
2	,5
1	,25

Nails.	*Decimals.*
3	,1875 .
2	,125 .
1	,0625

Measure.

Liquid. | *Dry.*

Integer.

1 *Gall.* | 1 *Quarter.*

Pints.	*Decim.*	*Bushel.*
7	,875	7
6	,76 .	6
5	,625	5
4	,5 . .	4
3	,375	3
2	,25 .	2
1	,125	1

Qs. Ps.	*Decim.*	*Pecks.*
3	,09375	3
2	,0625 .	2
1	,03125	1

Decimals.	*Qrs.Pecks.*
,023437	3
,015625	2
,007812	1

Decimals.	*Pints.*
,005859	3
,003906	2
,001953	1

ERRATA.

In the Advertiſement, *r.* Optician.

Page 7, the Quote ſhould be ,0084971334, *&c.*

P. 19. Exam. I. the Product by $\frac{4}{7}$, is 927940$\frac{3}{7}$.

P. 44, *l.* laſt but one, *dele* ſubtracted.

P. 124, Prop. XVII. in the Rule, *l.* 1. *r.* of the Circle *l.* 2, *dele* a.

P. 128, In finding the Segments Areas in this Ex. there is an Omiſſion of Multiplying by the half Chord; therefore *P.* 129. *l.* 5. *r.* $2,56 \times \frac{6}{2} = 7,68$, the Area; and *l.* 18. *r.* $2,2 \times \frac{4}{2} = 4,4$, the Area. Then $21,75 + 12 + 7,68 + 4,4 = 45,83$ Square Chains. *l.* 21. *r.* 4 Acres, 5 Square Chains, 13 Square Poles, *&c.*

P. 144, *l.* 20. *r.* add the ſeveral, *&c.*

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